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COHERENT GENERATION OF THE TERRESTRIAL KILOMETRIC
RADIATION BY NON-LINEAR BEATINGS
BETWEEN ELECTROSTATIC WAVES

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Radiation by Non-Linear Beatings
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ABSTRACT

The propagation of electrostatic plasma waves in an inhomogeneous and magnetized plasma is studied analytically. These waves, which are driven unstable by auroral beams of electrons, are shown to suffer a further geometrical amplification while they propagate towards resonances. Simultaneously their group velocities tend to be aligned with the geomagnetic field. Then it is shown that the electrostatic energy tends to accumulate at, or near ω_{LH} and ω_{UH}, the local lower and upper hybrid frequencies. Due to this process, large amplitude electrostatic waves with very narrow spectra should be observed near these frequencies at any place along the auroral field lines where intense beam driven instability takes place. These intense quasi-monochromatic electrostatic waves are then shown to give rise by a coherent non-linear three wave process to an intense electromagnetic radiation. Depending upon the ratio ω_{pe}/ω_{ce} between the electron plasma frequency and the electron gyro-frequency the electromagnetic wave can be radiated in the ordinary mode (at ω_{UH}) or in the extraordinary mode (at $2\omega_{UH}$). As far as the ratio ω_{pe}/ω_{ce} tends to be rather small it is shown that the most intense radiation should be observed at $2\omega_{UH}$ in the extraordinary mode.

INTRODUCTION

All magnetized planets seem to be intense radio sources, emitting in the

decametric range for Jupiter (Warwick 1961, Carr and Desch 1976) in the hecto-

metric range for saturn (Brown 1975), and in the kilometric range for the Earth

(Gurnett 1974, Alexander and Kaiser 1976). The temperature of brightness of these

sources are so large that any classical incoherent radiation process is irrelevant.

At least for the Earth and Jupiter it is widely admitted that this radio-emission

process takes place in the regions of their magnetospheres where in-

tense beams of electrons are known to exist. Thus it is clear that

the electromagnetic radiation is directly or indirectly driven by a

beam plasma instability. For non-relativistic beams of electrons most

of the energy driven by this beam plasma instability is electrostatic.

Thus the problem is to convert this electrostatic energy into electro-

magnetic radiation. The total peak power of Auroral Kilometric

Radiation (AKR hereafter) was estimated by Gurnett to be 10^9 W. which

can be 1% of the total energy available during auroral phenomena.

As far as the power of electrostatic noise should be less than the

power involved in auroral phenomena, it is required that the conversion

process have an efficiency of the order of at least 1%. It seems

that this rules out any linear conversion process as those proposed by Oya (1974)

and Benson (1975). Thus, what is required is a very efficient non-

linear conversion process. Some non-linear theory were already proposed,

we will comment briefly on two of them. In the work of Palmadesso et al. (1976) electrostatic high frequency waves, driven unstable by an energetic electron beam, are coupled with ionic fluctuations. The required level of ionic fluctuation is very high ($\delta n_i/n_i \approx 0.3$). If such high level ionic fluctuations exist, they should be localized in the return current regions as shown by Kindel and Kennel, (1975) that is where low energy upgoing electrons are observed. These return currents are known to take place outside the region where downgoing primary electron beams are observed. Thus the coupling between high frequency electrostatic waves and ionic fluctuations should not be very strong. Furthermore the authors of the quoted works have used in an unstable situation the theory of Dawson and Oberman (1962) which was demonstrated to hold for damped plasma waves. The transposition of this theory to the unstable case seems to be irrelevant. Galeev and Krasnosel'skikh (1976) have proposed a different interpretation. They argued that due to the large level of electrostatic fluctuations at the plasma frequency there is a tendency to cavitation. Then they considered that each such caviton radiates a coherent electromagnetic wave at $2\omega_{pe}$. But, as far as they should be a large number of such cavitons inside an auroral electron beam, one cannot expect that the radiation of this set of cavitons is still coherent. Even so, the maximum efficiency of their process seems not to be large enough to account for the experimental requirements.

Melrose (1976) has discussed a generation mechanism for A.K.R. where escaping electromagnetic waves are directly driven unstable by beam plasma instability. In principle such a mechanism is tempting since it is a direct process, but the obtained threshold condition : $\beta_{\perp_0}^2 \gtrless \beta_{\parallel_0}$ would imply a very large thermal anisotropy $T_\perp / T_\parallel \gtrsim 30$ in the beam frame. In absence of experimental confirmation, at least theoretical arguments explaining how such a large anisotropy could be achieved over auroral region, would be required.

We propose here a different interpretation where the electromagnetic raradiated is generated through a coherent beatings between electrostatic waves. The required coherency being due to the large enhancement of the amplitude of the electrostatic waves when they propagate towards resonances.

In what follows we will focus on the AKR. The relevance of the present theory to the explanation of other planetary radio-emissions will be examined elsewhere.

PROPAGATION CHARACTERISTICS OF THE PRIMARY ELECTROSTATIC WAVES

According to Green et al(1977), AKR is generated along auroral field lines in the regions where the plasma frequency ω_{pe} is less or of the order of the electron gyrofrequency ω_{ce}. Then provided that we have still $\omega_{pi} \gg \omega_{ci}$ where the subscript i stands for ions, the dispersion relation for electrostatic waves writes in the zero temperature limit:

$$D(\omega,\vec{k}) \equiv 1 - \frac{\omega_{pe}^2}{k^2} \left[\frac{k_{\parallel}^2}{\omega^2} + \frac{k_{\perp}^2}{\omega^2-\omega_{ce}^2}\right] - \frac{\omega_{pi}^2}{\omega^2} = 0 \qquad (1)$$

As long as $\omega/k_{\parallel} \gg V_{the}$ where V_{the} is the thermal velocity of the so-called "cold plasma", and $n_c \gg n_b$, n_c and n_b being respectively the cold plasma and beam densities, the expression (1) is valid. Of course very far from the earth where n_w the warm plasma density can be larger than n_c, thermal effects would be essential giving rise to Bernstein waves having $\omega \simeq (n + 1/2)\omega_{ce} < \omega_{UH}$. The coherent non-linear process that we will discuss later can also be pertinent in such situations but we will not examine this in the present paper.

Let us discuss now the dispersion relation (1). There are two propagation windows: $\omega_{LH} < \omega < \min (\omega_{pe}, \omega_{ce})$ and $\max (\omega_{pe}, \omega_{ce}) < \omega < \omega_{UH}$. Near from ω_{pe} and ω_{ce}, $k_{\parallel} \gg k_{\perp}$. The modulus of the wave number does not intervene in (1) thus $\vec{k} \cdot \vec{V}_g = 0$; since $k_{\parallel}$ is fixed by the resonance condition $|V_g|$ is well determined, even in the zero temperature limit. Then the introduction of a static magnetic field avoid the usual degeneracy of Langumir waves. From the previous relation we deduce that the group velocity is aligned with the static magnetic field $\vec{B}_o$ when $\omega \simeq \omega_{LH}$ or

ω_{UH}. On the other hand $\vec{V}_g \perp \vec{B}_o$ when $\omega \simeq \omega_{pe}$ or ω_{ce}.

In a situation where beam-plasma driven instability takes place only those electrostatic waves which have their parallel phase velocities in the same direction as the beam are amplified. As stated earlier, waves having $\omega \simeq \omega_{pe}$ or ω_{ce} essentially propagate across the magnetic field which implies that they rapidly leave the auroral tube of force, that is the beam region. Thus what we will discuss now is the behavior of waves having frequencies far enough from ω_{pe} and ω_{ce}. An unstable wave, in the lower frequency window, has its parallel group velocity directed towards the earth, consequently it propagates towards regions where ω_{LH} increases, thus the ratio $\omega/\omega_{LH}(Z)$ (Z being the distance along field lines) decreases. Simultaneously its group velocity becomes more and more aligned with $\vec{B}_o$. On the other hand, an unstable wave having: $\max(\omega_{pe}, \omega_{ce}) < \omega < \omega_{UH}$ propagates outwards since its parallel group velocity is directed away from the earth. Then the ratio $\omega/\omega_{UH}(Z)$ decreases while $\vec{V}_g$ becomes more and more aligned with $\vec{B}_o$. Then in both cases the energy of plasma waves propagates towards resonances, while their group velocities tend to be aligned with $\vec{B}_o$. During this propagation, as long as the WKB approximation holds, $k_\perp$ is constant and consequently $k_{\parallel}(Z)$ has to decrease in order to satisfy locally (1). The medium being essentially stationary it is clear that ω is constant. Then, as the plasma waves propagate towards resonance their parallel phase velocities increase while their parallel group velocities decrease (since $V_{g\parallel} \propto k_{\parallel}$) and simultaneously $|V_{g\perp}/V_{g\parallel}|$ decreases.

Depending upon the ratio $k_{||o}/k_{\perp o}$ between their parallel and perpendicular wave numbers the electrostatic waves can remain inside the beam region or rapidly leave it. Then it is important to evaluate the critical value $(k_{||o}/k_{\perp o})_{cr}$ corresponding to the transition between these two regimes. Let us first discuss this problem for the upper propagation window where the ratio $k_{||}/k_{\perp}$ can change over distances much shorter than it does in the lower one. Starting with a wave having ω, $k_{||o} = k_{||}(Z_o)$, $k_{\perp o} = k_{\perp}(Z_o)$ it will reach resonance at $Z = Z_o + \Delta Z$ where $\omega_{UH}^{i}(Z_o + \Delta Z) = \omega$. Let R_b be the radial size of the beam, if $\Delta Z/V_{g||o} \gtrsim R_b/V_{g\perp o}$ the wave will still be inside the beam when it reaches resonance. This condition also writes $k_{||o} \Delta Z = k_{\perp o} R_b$. Expanding $\omega_{UH}(Z_o + \Delta Z) = \omega_{UH}(Z_o) [1 + \beta \Delta Z]$ where $\beta \approx 3/R$, and using (1) we find:

$$\Delta Z = (k_{||o}/k_o)^2 \; (\omega_{peo}^2 \; \omega_{ceo}^2 \; R) \, / \, (6 \; \omega_{UHo}^4) \qquad (2)$$

Combining this expression and the relation $k_{||o} \; \Delta Z = k_{\perp o} \; R_b$ we get:

$$(k_{\perp o}/k_{||o}) \; [(k_{\perp o}/k_{||o})^2 + 1] = (\omega_{peo}^2 \; \omega_{ceo}^2 \; R) \, / \, (6 \; R_b \; \omega_{UHo}^4) \qquad (3)$$

Setting $R_b \approx 10^5$ m and hence $R_E/6R_b \simeq 10$ we obtain for $\omega_{ce} = 4\omega_{pe}$, $R=2R_E$: $(k_{||o}/k_{\perp o})_{cr} \simeq 1$, which corresponds to $\omega \approx (\omega_{ce} + \omega_{pe})/2$. Similar calculations performed near ω_{LH} give $(k_{||o}/k_{\perp o})_{cr} \simeq 2.4 \times 10^{-2}$ which corresponds to a wave starting with $\omega/\omega_{LHo} \simeq 1.4$.

LINEAR INSTABILITY OF BEAM DRIVEN PLASMA WAVES

We will now discuss the convective amplification of the electro-static waves which are excited by the auroral electron beam. The problem under discussion in this section was worked out numerically by Maggs(1976). However, as far as he was interested in the production of the auroral hiss he has not discussed the convective amplification in the upper propagation window. Furthermore, for the purpose of our future discussion we need analytical (even though less precise) expressions.

Let us let $E \propto \exp i \left\{ \int k_{||} dz + \vec{k}_\perp \cdot \vec{r} - \omega t \right\}$, $k_{||} = Jm(k_{||})$ and $G = - \int k_{||} dz$, $k_{||}$ is the spatial growth rate and G the effective gain along a ray path. f_b is the electron beam distribution function, it is isotropic and centered around V_b, its bulk velocity; its thermal velocity is $V_b \cdot f_{th}$, 0, U_{th} are the corresponding quantities for thermal electrons. Provided that $f_{th}(V_b - U_b) < f_b(V_b - U_b)$ one can safely describe the thermal plasma by a Dirac "function". A discussion of the intermediate case where the beam is not completely detached from thermal electrons can be found in Maggs. Then, assuming a cold plasma back-ground with $n_c \gg n_b$ we obtain by using standard methods the contribution of the Cerenkov pole to the growth rate:

$$ k_{||} \simeq + \frac{\pi^2}{n_c} \frac{\omega_{pe}^2}{k_{||}} \frac{\omega^2}{\omega^2 - \omega_{pe}^2} \left\{ \int dV_\perp^2 \, J_0^2 \left(\frac{k_\perp V_\perp}{\omega_{ce}} \right) \frac{\partial f_b}{\partial V_{||}} \right\}_{V_{||} = \frac{\omega}{k_{||}}} \tag{4} $$

where J_0 is a Bessel function. When $k_\perp \gg \omega_{ce}/U_b$ the contribution to the growth is very small; so that we have restricted ourselves to the case $k_\perp \lesssim \omega_{ce}/U_b$ which will be checked a posteriori. Then $J_0^2 \simeq 1$, and the expression (2) is very simple. What we have to do now is to evaluate $\int k_\parallel \, dz$. Let us look first at the upper propagation window and assume that $\omega_{ce} > \omega_{pe}$. For the other cases: $\omega_{ce} < \omega_{pe} \lesssim \omega < \omega_{UH}$ and $\omega_{LH} < \omega \ll \min(\omega_{pe}, \omega_{ce})$ the calculations are very similar and we will solely mention the results. From (1) we get:

$$\frac{\omega^2}{k_\parallel^2(Z)} = W^2 \frac{\omega^2 - \omega_c^2(Z)}{\omega_{UH}^2(Z) - \omega^2} \tag{5}$$

If ω_{pe} is significantly less than ω_{pe}, then, $(\omega^2 - \omega_{pe}^2)/\omega_{ce}^2 \approx 1 \gg (\omega^2 - \omega_{ce}^2)/\omega_{ce}^2$ or $(\omega_{UH}^2 - \omega^2)/\omega_{ce}^2$ these last two quantities being less than $\omega_{pe}^2/2\omega_{ce}^2$. Then the variation of ω_{pe}^2 can be neglected. Expanding the gyrofrequency $\omega_{ce}(Z) = \omega_{ceo}[1 + \beta(Z-Z_0)]$ we get from (5):

$$\frac{dz}{dV_{\varphi\parallel}} \, dV_{\varphi\parallel} = - \frac{V_{\varphi\parallel} \, dV_{\varphi\parallel}}{2\beta} \frac{\omega_{pe}^2}{\omega_{ceo}^2} \frac{W^2}{[W^2 + V_{\varphi\parallel}^2]^2} \tag{6}$$

Combining (2) and (4) and setting $F_b(V_\parallel) = \int f_b(V_\parallel, V_\perp) \, dV_\perp^2$ we obtain:

$$G = - \int_{Z_0}^{Z_1} k_\parallel \, dZ = \frac{\pi^2}{2n_e} \frac{\omega_{pe}^4}{\omega^2 - \omega_{pe}^2} \frac{\omega}{\omega_{co}^2} \beta \int_{V_{\varphi_0}}^{\infty} g(V_{\varphi\parallel}, W) \frac{\partial f_b}{\partial V_{\varphi\parallel}} \tag{7}$$

where $g(V_{\varphi\parallel}, W) = g(V_{\varphi\parallel}, W) = W^2 V_{\varphi\parallel}^2 [W^2 + V_{\varphi\parallel}^2]^{-2}$ and Z_1 corresponds to $k_\parallel \to 0$, that is to $\omega_{UH}(Z_1) = \omega$. Consequently, due to the inhomogeneity along field lines

the parallel phase velocity explores the beam from $V_{\varphi 0}$ to infinity. If

$V_{\varphi 0} > V_b$ the integral will be negative since $\partial F_b / \partial V_\varphi < 0$, then $G < 0$.

Thus it is needed that (for a beam symmetric with respect to V_b)

$V_{\varphi 0} \leqslant V_b - U$. But this is not sufficient, if g was constant we would then

get $G = 0$. It is easy to see that as long as $W < V_b$ the growth will

be larger than further damping. In order to evaluate the integral in

(7) we have to specify F_b, which is a difficult task since it is poorly

known. The simplest way is to use a square-ended beam distribution:

$F_b = (n_b/2U) \left\{ H [V_{\varphi \parallel} - (V_b - U)] - H [V_{\varphi \parallel} - (V_b + U)] \right\}$ where H is

the Heaviside "function". Then we obtain to the lowest order in

U/V_b (which must be a small parameter) :

$$G = \pi^2 \frac{n_b}{n_c} \frac{\omega_{pe}^4}{(\omega^2 - \omega_{pe}^2)} \frac{\omega}{\omega_{ceo}^2 \, \beta \, V_b} \frac{(1 - W^2/V_b^2)}{(1 + W^2/V_b^2)^3} \qquad (8)$$

It is clear from (8) that we must have $W^2 < V_b^2$ in order to get an

amplification. If we start at some point Z_0 with $\omega/k_{\parallel 0} \simeq V_b - U_b$,

the preceeding condition can be written: $W^2/V_b^2 \approx k_\parallel^2 W^2/\omega^2 \approx$

$(k_{\parallel 0}^2/k_{\perp 0}^2)[(\omega^2 - \omega_{peo}^2)/\omega^2]$. On the other hand, as mentioned above, it is

necessary that $(k_{\parallel 0}^2/k_{\perp 0}^2) > (U_b/V_b)^2$ otherwise finite larmor radius

effect would begin to decrease the size of the gain factor. Combining

these conditions we get $(U_b/V_b)^2 < (k_{\parallel 0}^2/k_{\perp 0}^2) < (\omega^2)/(\omega^2 - \omega_{peo}^2)$ which

seems reasonable since $\omega^2/(\omega^2 - \omega_{peo}^2) > 1$ and $(U_b/V_b)^2 \simeq 0.2$: See the observations

resumed in figure 2 of Maggs(1976). It is worthwhile to notice that the above

condition is also compatible with (3) which insures that most of the

wave energy remains inside the beam and hence can be convectively ampli-

fied by it.

In order to have an order of magnitude for G we have set $(W/V_b)^2 \simeq 1/2$, $\omega_{pe} \ll \omega_{ce}$ so that $\omega^2 - \omega_{pe}^2 \approx \omega_{ce}^2$, $V_b = 10^7$ m/s, $\beta = 3/R$ and $\omega = 2\pi \times 10^5$ Hz, $R = 2.5 R_E$. Then

$$G = 2 \times 10^5 \frac{n_b}{n_c} \left(\frac{\omega_{pe}}{\omega_{ceo}} \right)^4 \tag{9}$$

Then with $\underbrace{n_b/n_c = 10^{-3}}$ and $\omega_{pe}/\omega_{ceo} \approx 2$ we get $G = 12.5$ which is a rather large factor. It should be said at this point that the choice of a square ended beam has increased the size of G as compared as it would have been for a more smooth beam distribution.

The discussion of the case $\omega_{ce} < \omega < \omega_{UH}$ is essentially similar one gets with the same square-ended beam distribution and $W_2^2 = (\omega^2 - \omega_{ce}^2)/k_\perp^2 \approx c^{\underline{te}}$:

$$G_2 = \pi^2 \frac{n_b}{n_c} \frac{\omega}{\beta V_b} \frac{W_2^2/V_b^2}{(1 + W_2^2/V_b^2)^2} \tag{10}$$

Now G is always positive, the maximum corresponds to $W_2^2 \sim V_b^2$; for the same reasons than those discussed above we will require that $(U_b/V_b)^2 < k_{\parallel o}^2/k_{\perp o}^2 < 1$ the last inegality now comes from (3) in the case $\omega_{pe} \sim 2\omega_{ce}$.

Finally, when $\omega_{LH} < \omega \ll \min(\omega_{pe}, \omega_{ce})$ we get also:

$$G_3 = \pi^2 \frac{n_b}{n_c} \frac{\omega}{\beta V_b} \frac{W_3^2/V_b^2}{(1 + W_3^2/V_b^2)^2} \tag{11}$$

But now $W_3^2 = \omega^2(\omega^2 - \omega_{LHo}^2)/k_{\|o}^2 \; \omega_{Lo}^2$. G_3 is smaller than G_2 since now, in G_3 the frequency of interest is: $\omega \sim \sqrt{2}\omega_{LH}$ instead of being $\omega \approx \omega_{UH}$. Consequently the convective amplification is less than in the upper frequency window.

GEOMETRICAL AMPLIFICATION OF ELECTROSTATIC WAVES NEAR THE RESONANCES

When an electrostatic wave propagates towards resonance, as said earlier, $k_{\parallel}$ and hence $V_{g\parallel}$ decrease in order to satisfy (1) for increasing ratios $\omega/\omega_{UH}(Z)$ or decreasing ratios $\omega/\omega_{LH}(Z)$. If we assume that $V_{g\parallel} E^2$ is conserved then E^2 increases and would even become infinite when $k_{\parallel} \to 0$ that is when $\omega \simeq \omega_{UH}$ or ω_{LH}. Of course then the WKB procedure fails, we have a turning point. Let us evaluate this in a more appropriate way. Near a turning point (1) becomes in the upper propagation window:

$$\frac{d^2\varphi}{dZ^2} + k_{\perp}^2 \varphi \frac{[\omega_{UH}^2(Z) - \omega^2]}{[\omega^2-\omega_{ce}^2(Z)][\omega^2-\omega_{pe}^2(Z)]} = 0 \qquad (12)$$

As we are interested in the behavior of the electrostatic potential φ near from resonance, we can assume that: $\omega^2-\omega_{pe}^2 \approx \omega_{ce}^2$ and that $\omega^2-\omega_{pe}^2 \approx \omega_{ce}^2$ and keep them constant. Then expanding the upper hybrid frequency we find: $\omega_{UH}(Z) = \omega_{UHO} [1 + \beta (Z-Z_0)]$ where Z_0 is our starting point. Then $\omega_{UH}^2-\omega^2 = \omega_{UHO}^2-\omega^2 + 2 \beta \omega_{UHO}^2 (Z-Z_0)$ setting that $\omega_{UH}(Z_1) = \omega$, we get:

$$Z_0 - Z_1 = (\omega_{UHo}^2-\omega^2)/2\beta\omega_{UHo}^2 \qquad (13)$$

Then, working with $\xi = k_{\parallel o}^{2/3}(Z-Z_1)(Z_0-Z_1)^{-1/3}$ instead of Z we can rewrite (12) in the form:

$$d^2\varphi/d\xi^2 - \xi \, \varphi = 0 \tag{14}$$

whose solution is an Airy function $Ai(-\xi)$. According to this solution the electrostatic potential φ increases when $Z \rightarrow Z_1$, that is when the wave propagates towards resonance. Using the results of White and Chen (1974) we get :

$$(\varphi_{max}/\varphi_o) \simeq 1.93 \, [k_{||o}(Z_o-Z_1)]^{1/6} \tag{15}$$

φ_o being the potential of the incident wave far from Z_1. This result is valid as long as the reflection coefficient is one. Then starting with $k_{||o} \sim k_{\perp o}$ and assuming that $\beta \approx 3/R$, $V_b \simeq 10^7$ m/s, $\omega_{ce} \simeq 2\omega_{pe} \simeq 2\pi \times 10^5$ Hz, $R = 2.5 \, R_E$, we get $(\omega_{UHo}^2-\omega^2)/\omega_{UHo}^2 \approx \omega_{pe}^2/2\omega_{ce}^2 \approx 1/8$ and then from (15): $(\varphi_{max}/\varphi_o) \approx 10$ and $(E_{max}/E_o) \approx (k_{\perp o}/\sqrt{k_{\perp o}^2/k_{||o}^2}) \, (\varphi_{max}/\varphi_o) \approx 7$. This is a large increase since $(E_{max}/E_o^2) \simeq 50$.

For lower frequencies, that is for waves which propagate towards increasing lower hybrid frequencies the situation is very similar to the previous case. By similar calculations we get: $(\varphi_{max}/\varphi_o) = 1.93 \, [k_{||o} (\omega^2-\omega_{LHo}^2)/(2\beta \, \omega_{LHo}]^{1/6}$. As mentioned above, only waves starting at very small values of $(k_{||o}/k_{\perp o})$ can reach resonance inside the beam then (E_{max}/E_o) is quite as large as previously (~ 7).

Consequently, electrostatic waves having $k_{||o}/k_{\perp o}$ small enough to reach resonance inside the beam are linearly amplified by beam driven instability and also geometrically amplified when they propagate towards resonances. Thus, the electrostatic waves spectrum should peak

near ω_{UH} and ω_{LH} at a given place along auroral field lines. At a given place the frequency width of these spectra are essentially fixed by the Airy function. A wave having $\omega + \delta\omega$ will reach resonance at $\omega_{UH} + \delta\omega_{UH}$ that is at a point $Z_1 + \delta Z_1$ where $\delta\omega_{UH}/\omega_{UH} = 3\delta Z_1/R_E$. We will fix δZ_1 by setting that the argument of the Airy function varies from -2 to +2. Then $\delta Z_1 \approx 4(Z_0 - Z_1)^{1/3} k_{\|o}^{-2/3}$. Consequently we get:

$$\frac{\delta\omega_{UH}}{\omega_{UH}} \simeq \frac{12}{R} \left[\frac{\omega_{UHo}^2 - \omega^2}{\omega_{UHo}^2} \frac{R}{6} \left(\frac{V_b}{\omega}\right)^2 \right]^{1/3} \tag{16}$$

where $k_{\|o} \sim \omega/V_b$. Setting as above $\omega \simeq \omega_{ce} \simeq 2\omega_{pe} \simeq 2\pi \times 10^5$ Hz, $R = 2.5 \, R_E$ $V_b = 10^7$ m/s. We get $(\omega_{UHo}^2 - \omega^2)/\omega_{UHo}^2 \simeq 1/8$ and $\delta\omega_{UH}/\omega_{UH} \simeq 1.2 \times 10^{-3}$. Thus $\delta\omega_{UH} = 2\pi \times 130$ Hz, which is a very narrow spectrum. Of course this only corresponds to the largest peak of the Airy function.

NON-LINEAR INTERACTION BETWEEN PLASMA WAVES

We want to generate, by non-linear beating between electrostatic waves an electromagnetic wave which can escape from the source region. Then, if the subscript 1 and 2 refer to the primary electrostatic wave while 3 stands for the electromagnetic one, we have to insure that:

$$\vec{k}_3 = \vec{k}_1 \pm \vec{k}_2$$

$$\omega_3 = \omega_1 \pm \omega_2 \tag{17}$$

$$D(\omega_i, \vec{k}_i) = 0$$

where D is the dispersion relation, i = 1, 2, 3. In order to produce an electromagnetic wave it is necessary that $|k| \sim \omega/c \ll |k_{1,2}| \sim \omega/V_b$. Consequently we must have $|k_{||1}| \approx |k_{||2}|$ and $|k_{\perp 1}| \approx |k_{\perp 2}|$. The first of these conditions is not easy to fulfil, for instance if we want to couple ω_1 and ω_2 and get $\omega_3 = \omega_1 + \omega_2$, $\vec{k}_{||3} = \vec{k}_{||1} + \vec{k}_{||2}$, $\vec{k}_{\perp 3} = \vec{k}_{\perp 1} + \vec{k}_{\perp 2}$ we must have $\vec{k}_{||2} \simeq - \vec{k}_{||1}$. Then, since amplified waves have their parallel phase velocities in the same direction as the beam, only one of these waves can be amplified. Of course, the second condition is easy to satisfy, provided that the radial size of the beam is large and the distribution of $k_{\perp 1,2}$ is isotropic around B_o. As we know from previous discussion that the electrostatic waves have large amplitudes near ω_{LH} and ω_{UH} any strong coupling between such waves is

a priori likely to occur. Furthermore, near from resonances $|k_{\|1}|$ and $|k_{\|2}|$ are already very small, hence the matching condition $\vec{k}_{\|3} = \vec{k}_{\|1} + \vec{k}_{\|2}$ will be easily fulfilled. Then, in all what follows we will assume that primary electrostatic waves are near resonance.

An important question must now be discussed. Depending upon the width of the spectrum of the primary waves the non-linear decay or mode coupling can be coherent, or incoherent.

In the incoherent case, the random phase approximation (RPA hereafter) must be used. Due to the averaging over random phases the coupling between incoherent waves is usually rather small. In a recent work Barbosa (1977) has studied in the RPA, the generation of the AKR by non-linear coupling between two electrostatic waves having $\omega \approx \omega_{UH}$. Then he found that the primary electrostatic waves should have at least 3V/m in order to generate the observed large powers of electromagnetic waves. Which is probably too large by at least one order of magnitude, (which means four orders of magnitude as far as the electromagnetic energy is concerned).

In the two previous sections we have shown that near from ω_{LH} and ω_{UH} the spectrum of electrostatic waves is very peaked. Then a coherent three wave process is likely to occur.

Such a conclusion is valid if the phase velocity variation remains small over a non linear growth time. This condition can be written $\gamma_{NL} \gtrsim \Delta\omega$. γ_{NL} is the non linear growth rate that one obtains assuming that the non-linear wave-wave interaction process is coherent. $\Delta\omega$, the phase variation is linked to the spreading of the linear growth rate of plasma waves ; expanding, up to the second order, the phase of the electrostatic wave we get

$$\Delta\omega/2\pi \approx \left\{ \partial^2/\partial\omega^2 \left[\int (\gamma_L/V_{g_\parallel}) dz \right] \right\}^{-1/2}$$

which corresponds to a phase variation of 2π.

We have shown in the previous part that, in the upper propagation window $\int (\gamma_L/V_{g_\parallel}) dz \simeq 12$, the characteristic width of the convectively unstable waves $(\delta\omega)$ is much smaller than the upper propagation band itself, say 1/5 of it. Then we get $\delta\omega \simeq (\omega_{ce}/5)(\omega_{pe}^2/2\omega_{ce}^2)$; with $\omega_{ce} = 3\omega_{pe}$ and $\omega_{ce} \simeq 2\pi \times 100\text{kHz}$ we get $\delta\omega/2\pi \simeq 1\text{kHz}$ and $\Delta\omega/2\pi \simeq 300\text{Hz}$.

Consequently we must have $\gamma_{NL} \gtrsim 2\times10^3$.

Let us now discuss the coherent three wave process, keeping in mind that the previous condition must hold. Two cases are possible :

$\omega_1 \simeq \omega_2 \simeq \omega_{UH}$ and $\omega_1 \simeq \omega_{UH}$, $\omega_2 \simeq \omega_{LH}$, the frequencies of the resulting electromagnetic radiation being respectively $\omega_3 \simeq 2\omega_{UH}$ and $\omega_3 \simeq \omega_{UH}$.

RADIATION AT $\omega_3 \approx \omega_{UH}$

The non-linear currents which produce the electomagnetic radiation can be obtained by combining the Maxwell and fluid equations. In a situation where $\omega_{pe} \approx \omega_{ce}$ such a calculation is a priori very lengthy. Nevertheless, since $\omega_1 \simeq \omega_{UH}$ and $\omega_2 \approx \omega_{LH}$, their wave numbers are nearly perpendicular to the static magnetic field; furthermore, as said above $|k_3| \ll |k_1|$, $|k_2|$. Then the problem is greatly simplified. Parallel and perpendicular non-linear currents have similar order of magnitude. But, since we need a freely escaping electromagnetic radiation, only the parallel current, which generates an O-wave, is of interest here. We get three coupled equations, but will only write the one which describes the production of the electromagnetic wave. Retaining only the largest contribution to the non-linear current we obtain:

$$\frac{\partial D_3}{\partial k_{\perp 3}} \frac{\partial E_3(\omega_3, k_3)}{\partial r} \simeq \frac{e}{m} \frac{k_{\|2}}{\omega_2} \frac{k_{\perp 3}}{k_{\perp 1}} \frac{E(\omega_1, k_1) \, E(\omega_2, k_2)}{\omega_3}$$

$$D_3 \equiv \frac{k_{\perp 3}^2 \, c^2}{\omega_3^2} - 1 + \frac{\omega_{pe}^2}{\omega_3^2}$$

(18)

where $\omega_1 \approx \omega_{UH}$, ω_2 near from ω_{LH}, $\omega_3 \approx \omega_{UH}$. Since the O-mode propagates essentially across the magnetic field we have expanded $D_3 \, E_3 \approx i \frac{\partial D_3}{\partial k_\perp} \frac{\partial E_3}{\partial r}$. Then using the following relation: $\omega_2/k_{\|2} \approx \omega_{pe}/k_{\perp 2} \approx \omega_{pe}/k_{\perp 1}$, which holds even when ω_2 is near (but not at) ω_{LH}, (18) becomes:

$$\frac{\delta E_3}{\delta r} \simeq \frac{e}{2mc^2} \frac{\omega_3}{\omega_{pe}} E_1 E_2^{\,*} \tag{19}$$

Let us assume that $|E_1|\ |E_2| \gg |E_3|$, then (19) describes the production

of the O-radiation by non-linear coupling of electrostatic waves. As

long as $|E_3|$ remains small one can neglect the two other equations which

describe the reaction of the electromagnetic wave on the production of

the electrostatic ones. Then setting $\frac{\delta E_3}{\delta r} \approx \frac{E_3}{\Delta r}$, r being the distance

measured in a direction perpendicular to B_o, and Δr the characteristic

length over which E_1 and E_2 have large amplitudes, which should be

of the order of the radial extension of an inverted V event, that is,

for $L \sim 2\text{-}3\ R_E$, $\Delta r = R_b \simeq 10^5$ m. Setting $E_1 \approx E_2 \approx 100$ mV/m, $\omega_3 \approx 3\,\omega_{pe}$

we get $E_3 \approx 3$ mV/m, which seems to be an accurate value for moderately large AKR

events in the source region. The largest AKR events as observed by

Hawkeye, in the 178 kHz channel have $E_3 \approx$ 10-20 mV/m if one takes care

of the size of the propagation cone and of the dependance in $(R_E/R)^2$ of

the power flux (see Green et al, 1977). Of course we also need the ampli-

tudes of the electrostatic waves which is difficult to know since measurements

in the presumed source region are lacking. Only recently, during low altitude

$(R \sim 1.7\ R_E)$ over the southern hemisphere in auroral regions electric (from

1.78 Hz to 178 kHz) and magnetic (from 1.78 to 562 Hz) dc fields were measured

on board Hawkeye 1. When there is some indication of a field aligned current

in the night sector on auroral field lines, most of the electric field channels

measured strong noise, (see fig.8 of Gurnett and Green 1978).

Nevertheless it is difficult to know what are the electrostatic wave amplitudes

for $\omega \simeq \omega_{UH}$. As said earlier the peak at ω_{UH} should be very narrow; since the

178 kHz channel of Hawkeye is also very narrow there is very little chance that

the 178 kHz channel be measuring the UHR.noise at the right time and place.
Furthermore, since there are no plasma frequency measurements, the plasma
frequency (and thus the upper hybrid frequency) cannot be determined preci-
sely. For the above reasons one cannot deduce from present Hawkeye measu-
rements the electrostatic waves amplitude and spectrum for $\omega \sim \omega_{UH}$ in
regions where inverted V events take place.

Netherveless in the lower frequency channel, there is a strong indication
that we have a large electrostatic noise peaked around ω_{LH}. The 1.78 kHz
and 5.62 kHz channels are often at or near saturation when there is some
indication of a field aligned current over auroral regions. This means
that near from ω_{LH} the amplitude should be of the order or larger than
40 mV/m which is not far from what was required by the previous calculation.

We must insure that the condition $\gamma_{NL} \gtrsim \Delta\omega$ holds. Since $\gamma_{NL} \approx C/R_b \approx 3000$
and $\Delta\omega \sim 2\times10^3$ (see the previous part) the previous condition is marginally
satisfied.

It is also possible to couple low and high frequency electrostatic waves
having $\omega_1 \simeq \omega_{LH}$ and $\omega_2 \simeq \omega_{UH}$ in order to produce an extraordinary wave with
$\omega_3 \simeq \omega_{UH}$. The coupling coefficient is even stronger than the previous one
at least when $\omega_{ce} > \omega_{pe}$. Nevertheless, such an electrostatic wave cannot
escape from the source because its frequency is below the R-X cut-off
except for very small ratios upe/ω_{ce} which are unlikely on the earth but
might be reasonable in Jupiter's magnetosphere).

For the above mentioned reason we will delete here this possibility
for AKR generation.

RADIATION AT $2\omega_{UH}$

Setting $\omega_1 \approx \omega_2 \approx \omega_{UH}$ for the primary electrostatic waves we get by non-linear beatings a large non-linear current perpendicular to both the electrostatic wave numbers and $\vec{B}_o$. The corresponding parallel non linear current is completely negligible. Hence the radiated

wave is polarized in the X mode. Since the interaction involved here is a kind of self-interaction between the electrostatic wave which approaches resonance and a reflected wave , the matching conditions $\vec{k}_{\perp 1} + \vec{k}_{\perp 2} \simeq 0$ $\vec{k}_{\parallel 1} + \vec{k}_{\parallel 2} \simeq 0$ are easily satisfied. Then we get:

$$\frac{\partial D_3}{\partial k_\perp} \frac{\partial E_3}{\partial r} \simeq \frac{e}{m} k_\perp \cdot \frac{E_1 E_2^*}{\omega_{pe}^2} \left\{ 1 + \frac{\omega_1^2 + \omega_1 \omega_3}{\omega_3^2 - \omega_c^2} \left[1 + \frac{\omega_{pe}^2 \; \omega_{ce}}{\omega_3(\omega_3^2 - \omega_{UH}^2)} \right] \right\}$$

$$D_3 = \left\{ \frac{k_\perp^2 c^2}{\omega^2} - 1 + \frac{\omega_{pe}^2}{\omega^2 - \omega_{ce}^2} \left[1 + \frac{\omega_{pe}^2 \; \omega_{ce}^2}{\omega^2(\omega^2 - \omega_{UH}^2)} \right] \right\} \Big|_{\substack{\omega = \omega_3 \\ k_\perp = k_{\perp 3}}}$$

$$(20)$$

Provided that $\omega_{pe} < \omega_{ce}$ the last term in the RHS of the first equation is small and this equation rewrites:

$$\partial E_3/\partial r \approx E_3/\Delta r \approx \frac{e}{mc^2} \left(\frac{2\omega_{UH}}{\omega_{pe}}\right)^2 |E_1|^2 . \qquad (21)$$

Setting $\Delta r \approx R_b \approx 10^5$ m, $\omega_{UH} = 3\omega_{pe}$, $E_1 = 50$ mV/m we get $E_3 \simeq$ 18 mV/m which is fairly typical of strong TKR events. According to the

gyrofrequency and plasma frequency model of Green et al.(1977) : $\omega_{ce} \gg \omega_{ce}$ over a wide range of altitude along auroral field lines; which increases significantly the coupling between electromagnetic and electrostatic waves as can be seen in (21). It should be noted that if one uses the same parameters as above namely $\omega_{UH} \simeq 3\omega_{pe}$ $E_1 \approx E_2 \simeq 50$ mV/m, $\Delta r \simeq 10^5$ m for both O-wave and X-wave generation. The latter is 25 more efficient as far as the energy is concerned.

It should be noted that the convective amplification which runs like n_b/n_c $(\omega_{pe}/\omega_{ce})^4$ is greatly reduced when ω_{pe} decreases furthermore the geometrical amplification which runs like $(\omega_{pe}/\omega_{ce})^{1/3}$ also decreases when ω_{pe}/ω_{ce} decreases. Then, too small ratios ω_{pe}/ω_{ce} would finally decrease the energy of the electromagnetic radiation. The optimum value for ω_{pe}/ω_{ce} greatly depends upon the value of n_b/n_c which is very poorly known. In any case, it seems that this optimum value is below the expected value of ω_{pe}/ω_{ce} hence the conclusion that a decrease of ω_{pe}/ω_{ce} increases the efficiency of the radiation process seems to be valid. It should be said also that too large amplitude for the electromagnetic wave will tend to saturate the emission process by pumping too much electrostatic energy.

CONCLUSION

We have studied analytically the convective amplification of elec-
trostatic plasma waves by beam-driven instability. In both the lower
and upper propagation windows, amplified waves propagate towards resonances
(ω_{LH} and ω_{UH}); simultaneously the modulus of their group velocities and
their angle with the geomagnetic field tend to decrease. Thus the convec-
tively amplified electrostatic waves suffer a further "geometrical"
amplification while they propagate towards resonances. Consequently
the spectrum of electrostatic waves in the regions where inverted-V events
take place should peak near ω_{LH} or ω_{UH}.

Due to the narrowness of the spectra of these primary waves the
non-linear three wave interaction process which leads to electromagnetic
wave generation is coherent. This is essentially the reason why the
efficiency of the transformation of electrostatic waves into electromagnetic
is so large. In principle the escaping electromagnetic radiation could
be either in the O-mode with $\omega \approx \omega_{UH}$ or in the X-mode with $\omega \approx 2\omega_{UH}$.
The respective efficiencies of the two preceeding processes are essentially
controled by the ratio ω_{pe}/ω_{ce} which is smaller than unity over auroral
regions for $R \approx 2 - 4\ R_E$ (see Green et al 1977). When it is so, the
generation of the X-mode at $2\omega_{UH}$ becomes very efficient and largely overcomes
the generation of the O-mode at ω_{UH}. The strong A.K.R. events (with ampli-
tudes $E \sim 20$ mV/m) can easily be generated by beatings between two electro-
static waves having narrow spectra and $E \sim 50$ mV/m for $\omega \simeq \omega_{UH}$.
Such an amplitude does not seem unrealistic especially if one takes care
of the geometrical amplification effect; 50mV/m at ω_{UH} can correspond to

10mV/m just after the linear amplification stage, that is for ω not too near from ω_{UH}.

The energy balance is discussed in the appendix.

One should keep in mind that the generation of electromagnetic radiation in the X-mode at or near ω_{UH} which was not considered here since it doesn't escape from the source region, could be an important component of the Jupiter Decametric Radiation (J.D.R.). This difference between A.K.R. and J.D.R. is essentially due to the very small ratios ω_{pe}/ω_{ce} that presumably exist in Jupiter's magnetosphere. Then, as shown by smith (1976), tuneling through the "stop zones" is very efficient. For very small ratios ω_{pe}/ω_{ce} one could even have $\omega_{LH} + \omega_{UH} > \omega_{RX}$ the cutoff of the R-X mode, in such a case one could directly produce an escaping X-mode by beatings between electrostatic waves having $\omega_1 \simeq \omega_{UH}$ and $\omega_2 \simeq \omega_{LH}$.

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APPENDIX

Let us assume that the auroral region can be modeled by a parallelepiped L x H x d where d is the latitudinal extension of the auroral region at a distance $R \simeq 2.5\ R_E$, L is the longitudinal extension of the A. K. R. source at the same distance and h is vertical size. We have used $L \simeq H \simeq 10^7$ m, $d \simeq 10^5$ m.

We want to evaluate the efficiency of the conversion of electrostatic waves into electromagnetic ones. We have seen that the generation of the X mode at $2\omega_{UH}$ is the more likely to occur, thus we will only consider it here, which simplify the matter since only the electrostatic wave energy near from ω_{UH} is to be considered. The power flux of electrostatic waves is $\varepsilon_o V_{g_\parallel} L\ d\ E_{e.s.}^2$. Near from resonance the optical geometry fails and $E_{e.s.}$ greatly increases ; thus we will evaluate the power flux far from resonance, that is before that the focussing effect associated with this resonance has increased $E_{e.s.}$ by a large factor. The production of the electromagnetic wave at a given frequency takes over a small region L x δz_1 x d, where δz_1 its vertical size is defined by the spatial extension of the Airy peak (see formula 16 and discussion above). We get $\delta z_1 \simeq 4\ (z_o - z_1)^{1/3}\ k_{\parallel o}^{-2/3}$, for $\omega \simeq \omega_{ce} = 2\omega_{pe} = 2 \times 2\pi \times 10^5$ Hz, $V_b \simeq 10^7$ m/s and $R \simeq 2.5\ R_E$ we get $z_o - z_1 \simeq R_E/12$ and $\delta z_1 \simeq 1.3 \times 10^3$ m.

Then power flux conservation writes :

$$(A.1) \qquad (1 - R) \; E^2_{e.m} \; = \; \alpha \; E^2_{e.s}$$

where $\alpha = V_{g_{\parallel}} \; d / \; \delta z_1 \; C$, R being the energy reflexion coefficient. The reflected part of the electrostatic energy is proportional to $R \; E^2_{e.s.}$ hence eq. 21 rewrites :

$$(A.2) \qquad E^2_{e.m} \; = \; M^2 \; R \; E^4_{e.s}$$

where M is the coupling coefficient. Then combining (A.1) and (A.2) we get :

$$(A.3) \qquad E^2_{e.m} \; = \; E^2_{e.s} \; (\alpha + 1 / M^2 E^2_{e.s})$$

For large enough values of $E_{e.s.}$ the assymptotic solution of (A.3) is $E^2_{em} = \alpha E^2_{e.s.}$ with the previous values of the parameters and $V_{g_{\parallel}} / C \simeq 1/30$ we get $\alpha \simeq 3$. The electromagnetic wave energy is comparable to the electrostatic wave energy (as taken far from resonance). The same calculation can be made at any distance from the earth within the source, so that the real width of the A. K. R. spectrum, as observed far from the earth, results from the integration over the vertical size of the source of the locally monochromatic radiation processes.