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ON THE BOUNDARY LAYER BETWEEN A FLOWING PLASMA AND A MAGNETIC FIELD. I - THE CONDITIONS FOR EQUILIBRIUM

by

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ABSTRACT

A qualitative study is made of a plane charge-neutral magnetopause separating a field-free plasma from a plasma-free magnetic field, in the case where the plasma is flowing parallel to the applied field. It is shown that the existence of small-scale equilibrium depends on boundary conditions which are governed by the large-scale structure of the system. On certain conditions, which are defined, equilibrium is possible at all flow speeds. For a class of structures characterized by approximately axial flow, these conditions cannot be satisfied if the plasma encloses the magnetic field, but can if the magnetic field encloses the plasma. They are satisfied most naturally if, in addition, the magnetopause has the form of a torus. While intended originally as a contribution to magnetospheric physics, this study may be more relevant to the high-β confinement of fusion plasmas.

CONTENTS

1. INTRODUCTION

Any theoretician who sets out to derive the small-scale structure ("microstructure") of the Earth's magnetopause, for the purpose of comparison with experimental data from spacecraft, is obliged to take account of two factors - among others - which complicate the problem very much : firstly, the presence of a magnetic field in the flowing solar-wind plasma ; secondly, the presence of a plasma in the Earth's magnetic field. Taken together, they make the problem so difficult that it is unreasonable to seek full kinetic-theoretical solutions at the present time. Instead, some less rigorous approach has to be adopted. For instance, in a recent study of field-line reconnection at the front of the Earth's magnetopause, Haerendel (1978) has made use of analogies with the theory of eddy diffusion in turbulent boundary layers. Heuristic approaches such as this are likely to dominate the field for many years to come.

But to the theoretician less concerned with immediate relevance to experimental space physics, and more with the general physics of sharp boundaries between different plasma and magnetic field regimes, there remains the more formal approach that consists of retaining the basic kinetic equations in their full rigour, while seeking exact solutions to relatively simple problems. Starting from some situation where the answer is known, the complicating factors are introduced first individually, then in pairs, and so on, until the required degree of sophistication is achieved. The present paper is the first step in an approach of this type.

The precise purpose of this paper is to show that exact solutions should be obtainable in a case that has resisted quantitative treatment up to now. The case in point is that of a plane charge-neutral boundary layer between a field-free plasma and a plasma-free magnetic field, when the plasma is flowing parallel to the applied field. Were it not for the two factors mentioned in our opening paragraph, this state of affairs would exist at the Earth's magnetopause, in the plane defined by the solar-wind velocity vector and the magnetic dipole axis (Fig. 1).

In the absence of plasma flow, exact solutions of the collisionless Boltzmann equation that represent such boundaries have been derived previously. The problem is how to extend these solutions, when plasma flow is introduced as a complicating factor.

Hitherto such a programme faced an apparently insuperable difficulty, namely a demonstration by E.N. Parker and I. Lerche that whenever the flow speed exceeds the random thermal speed of the plasma ions, as it does over most of the magnetopause, no boundary layer can exist in static equilibrium. This demonstration is examined in the present paper, and is found to be partly valid, but not generally so. Equilibrium can indeed exist, though only on certain rather special conditions. This finding opens the path towards quantitative study of the structure and stability of the equilibrium boundary layers.

Some of the conditions for equilibrium refer to the properties of the plasma and of the magnetic field far from the layer, on either side of it. Mathematically speaking, they are what are normally called "boundary conditions", but use of this term in the present context might lead to confusion with the conditions existing in the boundary layer itself. Hence we shall refer to them instead as "conditions in the limits". Also we shall often use the term "magnetopause" as a synonym for "boundary layer", denoting any region of abrupt transition between markedly different states of the plasma and of the magnetic field, without necessarily referring to a planetary magnetopause.

The plan of this paper is as follows. Section 2 provides some background, by outlining previous work on the structure of the layer when the plasma is stationary. Section 3 recapitulates Parker and Lerche's case against the possibility of the boundary being in equilibrium when the plasma is flowing rapidly. Their reasoning is criticized in Section 4, leading to Section 5 where conditions are defined under which equilibrium can exist whatever the speed of flow. In Sections 6 and 7 these conditions are applied to studying the possible existence of equilibrium in various situations which, though very much idealized, nevertheless resemble certain space or laboratory systems ; toroidal systems prove to be of special interest. The eventual application of these findings to controlled fusion is discussed briefly in Section 8. Finally Section 9 summarizes the conclusions of the paper.

Detailed study of the equilibrium boundary layers is left to later papers, of which the companion paper II is the first.

SI units are used in these two papers. Vectors are symbolized by letters in bold faced type, their axial components and moduli by the corresponding symbols in ordinary type : thus $\mathbf{A} \equiv (A_x, A_y, A_z)$ and $A \equiv |\mathbf{A}|$.

2. LAYER STRUCTURE WITH A STATIONARY PLASMA

We begin by examining the equilibrium that exists in the absence of plasma flow. Our starting-point is the simple 1-dimensional boundary layer illustrated in Fig. 2. We take a right-handed set of rectangular coordinate axes Oxyz, with its origin O somewhere in the layer, and with the axis Ox parallel to the magnetic field. Oz is perpendicular to the boundary, and this is the sole direction in which the magnetic induction **B**, together with the electron density N_e and the ion density N_i, are supposed to vary. The plasma consists of electrons and of singly-charged positive ions of one species only ; collisions are neglected.

The conditions in the limits of very large negative or positive values of z are as follows :

$$\underline{\text{At } z = -\infty} \qquad \mathbf{B} = (0,\ 0,\ 0) \qquad N_e = N_i = N_o \tag{1a}$$

$$\underline{\text{At } z = +\infty} \qquad \mathbf{B} = (B_o,\ 0,\ 0) \qquad N_e = N_i = 0 \tag{1b}$$

We add the restriction that, in the former limit, the distribution functions of the electrons and of the ions should both be Maxwellian ; to simplify matters, we shall suppose them also to be at the same temperature, though this is not essential. The condition for pressure balance, namely

$$B_o^{\,2}/2\,\mu_o = 2\,N_o\,k\,T \tag{2}$$

where μ_o is the permeability of free space and k is Boltzmann's constant, relates the field B_o to the density N_o and the temperature T, so only two of these three parameters can be fixed arbitrarily. Finally, any electric field in the plasma will be assumed to arise from charge separation (i.e. polarization) only, so it must necessarily be parallel to Oz ; moreover, we shall assume later that this field is zero everywhere.

The structure of such a layer has been discussed by many authors, notably Grad (1961) and Laval and Pellat (1963). Their work has been summarized, in the context of magnetospheric physics, by Willis (1971, 1972, 1975, 1978) and Phelps (1973), and in the context of fusion plasma physics by Spalding (1971) and Haines (1977). Between them, these reviews contain references to most of the original papers, which should be consulted for the details of the various solutions ; here we shall limit our study to a small subset of these solutions, defined below.

But first we should mention how the different types of solution can be classified. In the boundary layer between the plasma and the magnetic field, the particle orbits are of two types : free orbits, on which the particles come from $z = -\infty$ and return there after reflection at the boundary ; trapped orbits, on which particles remain in the boundary layer indefinitely under the influence of the magnetic field, eventually aided by the polarization electric field. The solutions can be classified according to whether they involve trapped particles (and of what types), and also according to whether they involve an electric field.

We shall only consider solutions with no electric field, and therefore with no space charge : $N_e = N_i$ everywhere. Such solutions necessarily involve trapped particles. We shall suppose that these are all electrons, and that they are cold, i.e. that their kinetic energies of thermal motion are so low compared to those of the free particles that they exert no appreciable kinetic pressure. In the Earth's magnetopause, these cold electrons could come from the ionosphere at the foot of the polar cusp (Parker, 1960). With trapped particles of this type, the solution is unique and corresponds to the thinnest boundary layer that can exist in the absence of a polarization electric field.

The structure of such a layer is easy to understand, because the free particles that enter it from the plasma follow trajectories governed by $\mathbf{B}$ alone. Fig. 3, which has been adapted from Fig. 4 of Willis (1975), illustrates the trajectories for electrons and ions incident normally on the boundary. After each particle has entered the boundary layer, it executes an approximate half-circle, turning with its gyro-radius, then returns to the plasma. Particles incident obliquely turn around in distances varying from zero to twice their gyro-radius, according to the direction of incidence. Thus the gyro-radius can be regarded as the average depth of penetration of the plasma particles into the magnetic field.

The root-mean-square (r.m.s.) gyro-radius for Maxwellian charged particles in a magnetic field of induction B_o is

$$r = mV/eB_o = (2\ k\ T\ m)^{1/2}/eB_o \tag{3}$$

where m is the mass of the particle, e is the absolute value of the electronic charge, and

$$V = \langle v_y{}^2 + v_z{}^2 \rangle^{1/2} = (2\ k\ T/m)^{1/2} \tag{4}$$

is the r.m.s. value of the component of the particle velocity in the plane perpendicular to the field ; the angle brackets denote an ensemble average. Thus, for equal temperatures, the gyro-radius is greater for the ions than for the electrons in the ratio $(m_i/m_e)^{1/2}$.

Because the free ions penetrate further into the field than the free electrons do, they tend to create a positive space charge within the layer, and hence a polarization electric field directed towards the plasma. This field is conducted along the magnetic lines of force, down the polar cusp, and finally into the E-region of the ionosphere (Fig. 1) ; here it is short-circuited, because collisions between charged and neutral particles allow electric currents to flow accross the magnetic field in this region. As a result, relatively cold ionospheric electrons are sucked up the lines of force and into the boundary layer, until the space charge is neutralized and the electric field disappears (Parker, 1967a). The time required for this process to complete itself, after any change in the state of the flowing solar-wind plasma, has been estimated as a few minutes at the most (Parker, 1967b). With no space charge, the overall thickness of the layer is of the order of the ion gyro-radius.

In the light of this knowledge of individual particle trajectories, we now introduce some symbols and define some quantities that will be helpful later on in describing how the densities of the different particle species vary within the layer. In general, the subscript f will refer to the free electrons, and the subscript t to the trapped electrons. Thus N_f is the free-electron density, and N_t is the trapped-electron density. The condition for charge neutrality is

$$N_f + N_t \equiv N_e = N_i \tag{5}$$

Let z_f and z_i be the values of z at which N_f and N_i respectively are equal to $N_o/2$, and set

$$\Delta z \equiv z_i - z_f \tag{6}$$

Let δ_f and δ_i be the average depths to which the free electrons and ions respectively penetrate into the magnetic field to form the boundary layer ; with the plasma stationary, they can be taken as roughly equal to the gyro-radii r_f and r_i. The overall thickness of the layer is $\delta \simeq \delta_i \simeq \delta_f + \Delta z$. Evidently the definitions of z_f, z_i, and Δz are precise, while those of δ_f, δ_i, and δ are rather vague because of the difficulty in deciding where the uniform plasma ends and the boundary layer begins. Fig. 4 is a rough sketch of the electron and ion density profiles, illustrating these various definitions.

We now turn from the particle densities to considering the magnetic field and the related electric current. Since we have assumed that there are no temporal variations, the Maxwell equation governing the spatial variation of the magnetic field through the layer is

$$\nabla \times \mathbf{B} = \mu_o \mathbf{j} \tag{7}$$

where **j** is the current-density vector. Taking axial components,

$$\frac{\partial B_x}{\partial z} = \mu_o\, j_y \tag{8a}$$

$$\frac{\partial B_y}{\partial z} = -\,\mu_o\, j_x \tag{8b}$$

With the plasma stationary, Equation (8b) vanishes, but we shall need it in Section 3 when discussing the effects of plasma flow. In Equation (8a), the quantity j_y is the <u>surface</u> current density in the y-direction, i.e. the current flowing that way per unit area of the Oxz plane. Integration of this equation with respect to z, taking into account the conditions (1) for B_x in the limits, yields

$$B_o = \mu_o \int_{-\infty}^{\infty} j_y\, dz = \mu_o\, J_y \tag{9}$$

J_y is the <u>line</u> current density in the y-direction, i.e. the total current flowing that way within the boundary layer, per unit distance in the x-direction. From Fig. 3 it appears that, within the layer, the electrons are moving in the negative and the ions in the positive y-direction, so they both make positive contributions to J_y. The following rough-and-ready argument shows that these two contributions are approximately equal. The peak value of the contribution of the free electrons (or ions) to the surface current density in the y-direction may be estimated by supposing that these incident plasma particles are deflected by the magnetic field until they are moving parallel to the boundary, without their number density being reduced. Hence

$$j_{yf,i} \simeq e\, N_o\, V_{f,i} = e\, N_o\, (2\, kT/m_{f,i})^{1/2} \tag{10}$$

The corresponding contributions to the line current density are approximated by multiplying these quantities by the distances to which particles of the two types penetrate into the magnetic field, which we have called δ_f and δ_i :

$$J_{yf,i} \simeq j_{yf,i}\ \delta_{f,i} \tag{11}$$

If we take these equal to the gyro-radii $r_{f,i}$ as given by Equation (3), then the particle masses disappear and - using Equation (2) - we get

$$J_{yf} \simeq J_{yi} \simeq 2\ N_o\ kT/B_o = (N_o\ kT/\mu_o)^{1/2} = B_o/2\ \mu_o \tag{12}$$

The sum of these two contributions gives J_y in agreement with Equation (9). None the less, it should be emphasized that the approximations (12) are rather crude.

So much for the structure of the boundary layer in the absence of plasma flow.

3. CASE AGAINST EQUILIBRIUM WITH A FLOWING PLASMA

We now face the problem of how the structure described above would be modified by the introduction of uniform bulk flow of the plasma in the x-direction, i.e. in the direction of the externally-applied magnetic field.

A trivial solution, in which the electrons trapped in the boundary layer flow with the same velocity as the free plasma particles, could be obtained from the solution described in Section 2 by imparting to it a uniform motion along the field (Stubbs, 1968). Since this amounts to no more than a change of coordinates, it does not alter the layer structure. We assume, on the contrary, that the trapped electrons do not take part in the flow.

According to certain models of the Earth's magnetosphere, in which the solar-wind magnetic field is neglected, this state of affairs exists at the magnetopause in the plane defined by the Earth's magnetic dipole axis and the Sun-Earth line (see Fig. 1). Here the assumption that the trapped electrons are stationary is justified by the fact that if they were not so they would simply return to the ionosphere, on the closed dayside field lines at any rate, which they are restrained from doing by the space charge of the flowing positive ions.

The problem was first studied in this context, and immediately a major difficulty arose : it appeared that when the plasma flow is supersonic (with respect to ion acoustic waves, which travel roughly at the ion thermal speed V_i), as it is over most of the Earth's magnetopause, then the boundary layer cannot exist in time-stationary equilibrium. This was the conclusion of a series of papers by Parker, in collaboration with Lerche (Parker, 1967a,b, 1968a,b,c, 1969 ; Lerche, 1967, 1968 ; Lerche and Parker, 1967, 1979) ; an error in Lerche's analysis was corrected by Su and Sonnerup (1971), without affecting the conclusion qualitatively.

The argument of Parker and Lerche involves considering the magnetic fields due to the currents in the boundary layer. When defining the problem, in the opening paragraph of this section, we specified that the plasma is flowing in the x-direction, parallel to the applied magnetic field. But in addition to this primary field created by some external source (the Earth's magnetic dipole, as it happens) there are secondary fields created by currents in the layer. In particular, there is now a component of the secondary field in the y-direction, perpendicular to the plasma flow velocity. Equilibrium is held to be impossible because of the interaction between the flow and this transverse magnetic field component.

We now present the argument, though first in a form slightly different from that given by Parker and Lerche ; the original version will be presented afterwards. The reason for presenting the two different versions is to show that some of the assumptions made originally are inessential.

Let us begin by considering Fig. 5A which, like Fig. 2, shows a section of the boundary layer in the Ozx plane, in the 1-dimensional approximation that assumes the local radius of curvature of the layer to be much greater than its thickness. Across the layer, the component B_x of the magnetic induction vector drops from the value B_o to zero. This transition may be viewed as coming about through the superposition of two fields : a uniform primary field $B_o/2$ filling all space ; a secondary field, due to the current sheet in the boundary layer, which is equal to $- B_o/2$ below the layer ($z \ll 0$) and to $+ B_o/2$ above it ($z \gg 0$). In the plasma below the layer, the secondary field due to J_y exactly cancels the primary field. In the plasma-free space above the layer, the secondary field reinforces the primary field, doubling it to be precise. Hence the field that needs to be applied externally, in order to support the pressure of the plasma, has only half the value given by Equation (2) ; the other half is generated by J_y. This view of how the boundary-layer current sheet keeps the field component B_x out of the plasma applies equally well whether the plasma is stationary, as in the case studied in section 2, or whether it is flowing parallel to Ox as we are supposing at the moment.

Now consider Fig. 5B, which shows a section in the Oyz plane. When the plasma is stationary, there is no component of magnetic field in this plane. But, in the presence of flow, the free ions in the boundary layer take part in this motion while the trapped electrons do not, according to our hypotheses. Hence there is a component of current in the x-direction within the layer. By reasoning similar to that which led to Equation (11) via (10), the contribution of the ions to the line current density is approximately

$$J_{xi} \simeq e\, N_o\, U\, \delta_i \tag{13a}$$

$$\sim J_{yi}\, (U/V_i) \simeq (B_o/2\,\mu_o)\, (U/V_i) \tag{13b}$$

where U is the plasma flow speed. The contribution $J_{xf} \simeq -\, e\, N_o\, U\, \delta_f$ due to the flow motion of the free electrons can be ignored because $\delta_f << \delta_i$, so if this is the only contribution that the free electrons make, it follows that $J_x \sim J_{xi}$ as given by Equation (13). In a vacuum, this current sheet would generate a component of **B** in the y-direction :

$$B_y(z = \mp\,\infty) = \pm\, \mu_o\, J_x \sim \pm\, \frac{1}{2}\, B_o(U/V_i) \tag{14}$$

In the second and third terms of this equation, the positive signs apply below the layer, and the negative signs above ; $B_y = 0$ somewhere in the middle of the layer. The question arises as to what happens to this field component in the presence of the plasma. Now, in the Ozx plane, the two contributions to the field in the plasma cancel one another exactly, as we saw in the preceding paragraph. Here in the Oyz plane, in contrast, there is no background field, so there is a problem as to how the field due to the boundary-layer current sheet can be kept out of the plasma. The only possibility is that it be held back by the pressure of the hot electrons. In other words, these electrons on free trajectories must produce, within the boundary layer, a line current density J_{xf} equal and opposite to J_{xi} as given by Equation (13), thus reducing J_x to zero and making B_y vanish everywhere. This they cannot do by their flow motion alone, as noted above, so we must invoke their much faster thermal motion. But we have already shown, in deriving Equation (12), that the maximum line current density that the free electrons can create by their random thermal motion, converted into a systematic drift motion through their encounter with the magnetic field, is about equal to that which the ions can create by the same means (i.e. it equals J_{yi}). Therefore so long as the flow is subsonic ($U < V_i$), implying $J_{xi} < J_{yi}$, there is some possibility of J_{xf} being large enough to cancel J_{xi}. But when the flow is supersonic, it follows that $J_{xi} > J_{yi}$, and the pressure of the free electrons can no longer withstand that of the y-component of **B** due to the flowing ions. Consequently these electrons are pushed back, uncovering more ions, which augment J_x and hence B_y, exacerbating the problem, and the boundary layer breaks up catastrophically.

In their most concise statement of it, Lerche and Parker (1967) present their argument in a form consistent with that set out above, but elsewhere their more detailed explanations involve the following differences. They assumed that, whatever the value of U, the total line current in the x-direction would be zero ; the ion current J_{xi} in the boundary layer would be closed by an equal and opposite electron current J_{xf} in the plasma. They believed that this state of affairs would arise automatically, because J_{xf} would be carried by plasma electrons accumulated "in a semi-trapped condition" at the surface of the uniform plasma, where it merges into the boundary layer (Parker, 1967a). They held that the semi-trapped electrons are equal in number to the free ions in the layer, and that J_{xf} arises from their bulk motion in the x-direction with the velocity **U**. The space charge due to the accumulated electrons is neutralized, in Parker and Lerche's view, by cold ions trapped on the magnetic field lines. These assumptions, which are illustrated by Fig. 1 of Parker (1967a) and by Fig. 5 of Willis (1975), ensure that $J_{xf} = - J_{xi}$ and hence that $J_x = 0$.

As a result, the conditions for B_y in the limits $z = \pm \infty$ differ from those that we assumed previously. Now $B_y = 0$ in these limits, instead of having the values given by Equation (14). These conditions imply that $B_y \sim - B_o(U/V_i)$ somewhere in the middle of the layer, more or less at the level z_f defined in Section 2.

This state of affairs is even less conducive to equilibrium, because the free electrons are called upon to resist four times more magnetic pressure than in the previous instance, and also because the free ions flowing in the boundary layer, across the field component B_y, experience a force $\mathbf{U} \times \mathbf{B}$ pushing them upwards and away from the main body of the plasma.

Thus Parker and Lerche's original argument differs from our version of it in respect of the assumptions about the component J_x of the line-current density, and about the conditions for the magnetic field component B_y in the limits. Developed quantitatively, the two versions would lead to slightly different estimates of the value of U above which equilibrium is impossible. But qualitatively the conclusions are the same : there can be no equilibrium when $U \gg V_i$.

4. CRITICISM OF THE CASE AGAINST EQUILIBRIUM

The case against the possible existence of equilibrium, as stated in Section 3, has flaws in it. The current J_{xi} does not close automatically in the way that Parker and Lerche suggested. It can close through the plasma, either wholly or partly, but need not do so. We are free to assume, as we did in our own version of Parker and Lerche's argument, that the electron current in the x-direction is negligible and hence that $J_x \simeq J_{xi}$; however, this does not change the conclusions qualitatively. The really crucial flaw in the argument is that the conditions for B_y in the limits are not yet specified. Among the various conditions that are physically possible, some allow equilibrium to exist at all flow speeds, even highly supersonic. We now justify these statements.

Parker and Lerche's model for the automatic closure of J_{xi} involved semi-trapped electrons, neutralized by trapped ions, at the interface between the uniform plasma and the boundary layer. If at all, the electrons might be expected to accumulate in this way when the boundary layer contains no trapped particles, in which case the free ions attract the free electrons through the intermediary of the polarization electric field. But the studies by Grad (1961) and by Laval and Pellat (1963) of boundary layers formed in the absence of trapped particles showed no tendency for the electrons to accumulate anywhere ; Fig. 3a of Willis (1972) is misleading in this respect. In point of fact, the densities of the free electrons and ions both decrease monotonically with increasing z, the electron density more rapidly than the ion density. Hence a charge-neutral boundary layer cannot exist without trapped electrons, but does not require trapped ions. Evidently the current J_{xf} is not carried in the exact way that Parker and Lerche believed it to be, and it is not necessarily equal to $- J_{xi}$.

Actually J_{xf} is carried in the same way as J_{yf}, by the process described in Section 2. It is driven by the gradient of the kinetic pressure of the free electrons, in conjunction with the transverse component of the magnetic field, which it also creates. The current and the field are mutually consistent. Equation (12) gives an approximate upper limit to the line current density that can be carried in this manner. This limit can be exceeded only if an electric field is applied to the plasma in the x-direction, in which case the current is no longer confined to the boundary layer but penetrates into the main body of the plasma, taking the magnetic field with it ; exactly how it does so is still a live issue (Lin and Dawson, 1978). The pressure of the magnetic field then exceeds the kinetic pressure of the free electrons, so the latter are repelled in the direction of negative z. This is what happens when $U \gg V_i$ and $J_x = 0$, according to Parker and Lerche. But their grounds for assuming $J_x = 0$ are fallacious, as we have just shown. We have no reason yet to assume this, any more than that $J_y = 0$. Had we made the latter assumption, we would have come to the absurd conclusion that equilibrium is impossible even with a stationary plasma. This is why, in presenting our own version of Parker and Lerche's argument, we felt free to assume that $J_x \neq 0$, and in particular that $J_x \simeq J_{xi}$.

However, as we saw in Section 3, the two versions of the argument lead qualitatively to the same conclusion. What - if anything - is wrong with both of them ?

The basic flaw in our version is that the Maxwell Equation (8b), when integrated with respect to z through the boundary layer, tells us to what extent the magnetic-field component B_y changes from one level to another, but says nothing about its absolute values. Integration of this equation introduces an arbitrary constant needing to be determined by other considerations. Specifically, we lack conditions for B_y in the limits $z = \pm \infty$, analogous to the conditions (1) for B_x. Indeed it will appear shortly that, when the plasma is flowing, even the conditions (1) may have to be modified.

In their original argument, Parker and Lerche assumed that $J_x = 0$, in which case integration of Equation (8b) yields $B_y(z = -\infty) = B_y(z = +\infty)$. There being no other source for B_y, they naturally supposed that its common value in these two limits was zero ; besides, any other value at $z = -\infty$ would have been inconsistent with the hypothesis of a field-free plasma. The rest of their argument then followed. However, their assumption that $J_x = 0$ was not justified.

In our version of the argument we assume $|J_{xf}| \ll |J_{xi}|$, so $J_x \simeq J_{xi}$. Integration of Equation (8b) then yields

$$B_y(z = +\infty) - B_y(z = -\infty) = -\mu_o J_x \tag{15}$$

This result is consistent with the field component B_y having the same modulus but opposite directions on the two sides of the boundary layer, as assumed in Section 3, as stated explicitly in Equation (14), and as illustrated in Fig. 5A. But it is equally consistent with there being no field in the plasma (i.e. with the condition (1a), according to which B_y as well as B_x is zero at $z = -\infty$), and a field component $B_y(z = +\infty) = -\mu_o J_x$ in free space. Then, since we have also assumed that J_{xf} is small, we conclude that $|B_y| \ll B_o$ at the level z_f, so the free-electron population is not subjected to a magnetic pressure which it cannot sustain, and equilibrium seems to be possible.

An apparent difficulty remains, however. If we assume that J_x has approximately the value given by Equation (13b), we find that $B_y(z = +\infty) \simeq -B_o(U/V_i)$. When $U > V_i$ it appears that $|B_y(z = +\infty)| > B_o$, so the external magnetic pressure exceeds the total plasma-kinetic pressure. But this pressure is experienced only by the ions, and it helps to push them back into the plasma. Within the boundary layer, the $\mathbf{U} \times \mathbf{B}$ force is now directed downwards, which means that the layer must be thinner when the plasma is flowing than when it is stationary ; in the notation of Section 2, we now have $\delta_i < r_i$. Hence, while Equation (13a) is still a valid approximation, Equation (13b) is wrong and must be replaced by

$$J_{xi} \simeq (B_o/2\,\mu_o)\,(U/V_i)\,(\delta_i/r_i) \tag{16}$$

while Equation (14) must be replaced by

$$B_y(z = +\infty) \simeq \frac{1}{2}\,B_o\,(U/V_i)\,(\delta_i/r_i) \tag{17}$$

The fact that $|B_y(z = +\infty)|$ cannot exceed B_o sets a rough upper limit on δ_i, depending on the plasma flow speed : $\delta_i < 2(V_i/U)\,r_i$. The reasoning that sets this limit is sound only so long as the limit is much greater than r_f, that is, so long as $U \ll V_f$: for larger values of U it needs refining. Thus, if it is recognized that the boundary layer shrinks as U increases, then there is no difficulty in accepting that B_y remains finite in the limit $z = +\infty$.

If this is so, then the condition (1b) for the magnetic field in this limit must be replaced by

$$\underline{\text{At } z = +\infty} \qquad \mathbf{B} = (B_x,\, B_y,\, 0) \tag{18}$$

where, by integration of Equations (8a) and (8b), we have

$$B_x(z = +\infty) = \mu_o\,J_y \tag{19a}$$

$$B_y(z = +\infty) = -\,\mu_o\,J_x \tag{19b}$$

Because there must still be an overall balance between the magnetic and plasma-kinetic pressures, it follows that

$$B_x^{\,2} + B_y^{\,2} = B_o^{\,2} \tag{20}$$

at $z = +\infty$, with B_o^2 given by Equation (2). This last result replaces Equation (9).

Summarizing, the case against equilibrium with a flowing plasma is flawed by uncertainties as to whether the ionic contribution to the component J_x of the line-current density is closed by an equal and opposite electronic contribution, and also as to the conditions for B_y in the limits. Within the latitude that these uncertainties leave, it appears that equilibrium might be possible at all flow speeds. Under what conditions remains to be seen.

5. CONDITIONS FOR EQUILIBRIUM WITH A FLOWING PLASMA

It now appears that the boundary layer can exist in equilibrium, irrespective of the plasma flow speed, so long as the free electrons are not subjected to a greater magnetic pressure than they can bear. This requirement can be met if J_{xf} stays below the upper limit defined in Section 4, and if the conditions for B_y in the limits $z = \pm \infty$ are favourable. These are the basic conditions for equilibrium, but they are useless until the following two questions are answered : what governs the value of J_{xf}, and what governs the conditions for B_y in the limits ? The conditions for equilibrium must be expressed in terms of whatever factors may govern them, for only then will it be possible to see whether they can be satisfied.

To answer these questions, we have first to admit that the 1-dimensional problem that Parker and Lerche considered, and which we also have been considering up to now, contains no features capable of specifying either the value of J_{xf} or the conditions for B_y in the limits. But if we recognize that this problem is an idealization of one that, on a much larger scale, is actually 3-dimensional, then we perceive that it is precisely this large-scale structure that governs the conditions for equilibrium. Via these conditions, the small-scale structure is related to the large-scale structure ("macrostructure"). To express them in the proper terms, we must examine the closure of the currents in the boundary layer, and also the closure of the magnetic fields that they create. Since these secondary fields are perpendicular to the currents, we must consider how the system varies both in the x-direction and in the y-direction. Thus, even if we are only interested in the microstructure of the layer, there is no escape from having to consider a complete 3-dimensional system, with non-divergent and mutually consistent currents and magnetic fields, so the problem is more complicated than it appears at first sight. But it is not unduly complicated, since once the conditions for equilibrium have been established and shown to be satisfied at the particular point on the magnetopause that we are interested in, we can forget the macrostructure and study the equilibrium microstructure as a problem in one dimension.

Consideration of the macrostructure also enables us to tie up some loose ends in Sections 3 and 4. When, in Section 3, we presented our verion of Parker and Lerche's argument, we stated that the magnetic field component B_x in free space could be though of as the sum of two equal contributions : a "primary" field due to some external source, and a "secondary" field due to the component J_y of the line-current density in the boundary layer. In Section 4, however, when we discussed how the component J_{xf} was carried by the free electrons, we thought of the associated magnetic field as being created entirely by this component. In a purely 1-dimensional system there is no way of distinguishing between these opposite viewpoints, but in a 3-dimensional system the distinction can be justified. The primary (or "applied") field is simply the field that would be created by the permanent magnets and current-carrying conductors in the system, in the absence of the plasma.

Unfortunately the conditions for small-scale equilibrium of the magnetopause cannot be stated in terms of the large-scale structure of the system in such a simple and general way. As regards the line current J_x, it is clear that J_{xi} must be able to close otherwise than through the existence of an equal and opposite J_{xf}, but it is hard to be more precise without knowing more about the system. Similarly, in order to arrive at any clear-cut statement about the conditions for B_y in the limits, it is necessary to consider a restricted class of systems. This approach is followed in the next two sections.

6. EQUILIBRIUM IN SYSTEMS WITH AXIAL FLOW

For the reasons given in Section 5, we are now led to consider the large-scale structure of a class of 3-dimensional systems in which a hot, isotropic plasma flows past a stationary, localized magnetic field at speeds possibly much greater than the ion thermal speed. The systems in question are simple in that they involve purely axial flow. By this we mean the following : (a) the system has an axis of rotational symmetry ; (b) nowhere is there any component of flow velocity in the azimuthal direction, around the axis ; (c) the component of velocity parallel to the axis has the same sign everywhere. Because of (b), any streamline of the flow lies in a plane that includes the axis. Because of (c), no streamline can form a closed loop. Although such systems are 3-dimensional in the sense that their plasmas and fields vary in any three mutually perpendicular directions, in fact there are only two independent spatial coordinates, namely distance along the axis and radial distance from it.

At an arbitrary point on the magnetopause, any plane containing the symmetry axis also contains the local x-axis. The value of J_{xf} can be determined by considering how the lines of electric current are closed in this plane, the main question being how J_{xi} is closed.

In any plane perpendicular to the symmetry axis, the cross-section of the magnetosphere has the form of an annulus. The local y-axis lies in this plane, and the magnetic lines of force are circles. The conditions for B_y in the limits can be determined by applying Ampère's law to a circuit either just inside or just outside the magnetopause.

For generality, we shall consider examples of two types of system with axial flow. In the first, the plasma surrounds the magnetic field : in the second, the field surrounds the plasma. In both cases, we shall assume that the magnetopause is charge-neutral.

In our first example, the plasma flows around and confines the field of a dipole, the axis of which is parallel to the mean direction of the flow. This situation may arise on the planet Uranus (Siscoe, 1971). It is illustrated in Fig. 6, which has been drawn from imagination. On the upwind polar axis, a cusp is opened by the kinetic and ram pressures of the flowing plasma (Kennel, 1973). Though particles from the plasma may be precipitated in this cusp, nevertheless we shall hypothesize that no net electric current can flow along the axis of the system. Granted this, the ion current J_{xi} flowing in the magnetopause boundary layer can be closed only by an equal and opposite electron current J_{xf}, and we have seen that this makes equilibrium impossible when $U >> V_i$.

The same conclusion could be reached by considering the conditions for B_y in the limits. Let us suppose that we are interested in the equilibrium of the magnetopause in the plane where its diameter is greatest (marked by a broken line in Fig. 6). Fig. 7A shows a section through the system in this plane, which is the Oyz plane. Now we apply Ampère's law to a circuit just inside the magnetopause (the dashed line in Fig. 7A). According to our hypothesis, this circuit encloses no current, and it follows that B_y is zero on the vacuum side of the magnetopause. This condition in the limit $z = + \infty$ was assumed by Parker and Lerche, so here their argument is sound. Moreover, in this rotationally-symmetric system, the $\mathbf{U} \times \mathbf{B}$ force that helps to disrupt the magnetopause, by pulling the ions out of it, is seen to be simply the pinch force acting on the current J_{xi}. To conclude, in the system considered, which - apart from the direction of the magnetic dipole - has many features in common with the Earth's magnetosphere, the magnetopause indeed has no equilibrium state, for the reasons that Parker and Lerche gave.

In the second general type of system that we wish to consider, the magnetic field surrounds the plasma. An example is shown in Fig. 8. It is that of a supersonic plasma jet flowing along the axis of a rotationally-symmetric magnetic field. Actually the jet would diverge slightly due to the transverse thermal motions of the plasma particles, a feature which has been neglected in the figure. The field is created by a current flowing in an external conducting ring. The plasma jet pushes the magnetic field out a central cylindrical region, bounded by a magnetopause which we again assume to be charge-neutral.

In this instance the determination of J_{xf} is more delicate, because the plasma jet is capable of carrying a current, so there is an open question as to whether or not it does so. One would be inclined to assume that there is no net current in the jet, since presumably the plasma source produces ions and electrons in equal numbers. Moreover, if the metal target on which the jet impinges (Fig. 8) is allowed to float electrically ($I = 0$), this ensures that equal fluxes of the two types of particle are extracted from the source and transported to the target in the jet. Assuming this, it follows that the current carried by the flowing ions in the magnetopause must return backwards through the ring, carried by the free electrons. As before, this mode of current closure precludes equilibrium.

However, since the plasma jet is capable of carrying a net current, the assumption that it does not is less of a hypothesis about the physics of the system than a decision to consider one particular instance out of a range of possible states that the system could occupy. We are free to decide otherwise. If the target is connected to the plasma source through a high-impedance generator of direct current, as shown in Fig. 8, then the jet can be made to carry any current that we please, within the limit discussed in Section 4. The source then emits more ions than electrons, or vice versa, depending on the sign of the current. This is perfectly feasible : the surplus particles simply return to the walls of the source. We shall take the case in which the jet carries a positive current $I = \pi\, d\, J_{xi}$, where d is the diameter of the magnetopause in the mid-plane of the current-carrying ring, and J_{xi} is measured in this plane.

In this particular state of the system, the current J_{xi} closes by flowing forwards along the surface of the jet to the target, then returning to the source through the external circuit. Accordingly $J_{xf} = 0$ in the mid-plane of the ring. As explained in Sections 4 and 5, the vanishing of J_{xf} enables equilibrium to exist, provided that, in addition, the conditions for B_y in the limits are favourable.

To derive these conditions we proceed as before, and apply Ampère's law to a circuit just inside the magnetopause (the dashed line in Fig. 7B). In the assumed state of the system, this circuit encloses no current ; the bulk flow velocities of the free electrons and ions are equal in the mid-plane of the ring. Hence B_y is zero inside the magnetopause, as before, but this is now the side on which the plasma lies.

Outside, in free space, B_y is given approximately by Equation (17). These are the conditions on which equilibrium exists (see Section 4). B_y does not exert any significant pressure on the free electrons. There is a $\mathbf{U} \times \mathbf{B}$ pinch force acting on the ions as before, but now this inward-directed force returns them to the plasma, and so compresses the boundary layer. Thus not only can be magnetopause exist in equilibrium, but one of the forces that destroyed equilibrium in the previous example now actually helps to maintain it.

The conditions for equilibrium that we have just defined are sufficient but not necessary. Obviously, equilibrium would not suddenly become impossible if the current I were not set exactly to the value of $\pi\, d\, J_{xi}$ in the mid-plane of the ring, with the result that J_{xf} were small but finite in this plane. Even when I has the correct value, J_{xf} is finite everywhere else. But since J_{xi} is less out of the mid-plane than in it, J_{xf} is everywhere positive, i.e. it has the same sign as J_{xi}. This state of affairs favours equilibrium. It would exist even in the mid-plane if $I > \pi\, d\, J_{xi}$. On the other hand, if $I < \pi\, d\, J_{xi}$ in the mid-plane, then J_{xf} would be negative in a certain region on either side of it, and there would be a risk of disequilibrium in that region even if $|J_{xf}| < J_{xi}$ everywhere. We are not yet able to calculate, as a function of U/V_i, the largest negative value of J_{xf}/J_{xi} at which equilibrium still exists. Our ambition has been limited to showing that a boundary layer, between a supersonically flowing plasma and a magnetic field applied in the direction of flow, can indeed exist in equilibrium ; this we have now done.

The objection might be made that equilibrium has been secured by unnatural means, namely external conductors and an external source of current. Our reply would be that the means in question are physically realizable, so their use in no way detracts from our claim to have shown that equilibrium can exist with a flowing plasma. The objection might have more substance if the magnetic fields produced by these current-carrying conductors were felt by the plasma. For this very reason, we have refrained hitherto from mentioning that equilibrium could be attained even in the system shown in Fig. 6, by installing on the central "planet" a constant-current generator, from which conducting wires run fore and aft, along the axis of the system, to make contact with the plasma upstream and downstream. With $I = \pi\, d\, J_{xi}$ as before, equilibrium is assured. The pinch force experienced by the flowing ions in the boundary layer is more than counterbalanced by the repulsion of the oppositely-directed current in the wires. The fact that the field due to this current is perpendicular to the direction of flow, whereas we have restricted our study to parallel applied fields, is another reason for excluding this type of system, which is known to fusion plasma physicists as a "hard-core pinch" (Rostoker, 1966). In the system of Fig. 8, however, the external conductors are supposed to be sufficiently remote from the plasma jet that their magnetic fields do not affect it, so the objection cannot be sustained on these grounds. None the less, we shall return to this point in Section 7.

To conclude the present section, let us summarize its main results. In systems with axial flow, equilibrium cannot exist at supersonic flow speeds when the plasma surrounds the magnetic field (Fig. 6), but it can when the magnetic field surrounds the plasma (Fig. 8). This difference appears to be due to the fact that the pinch force acting on the ions in the boundary layer is always directed inwards, towards the axis of the system.

7. EQUILIBRIUM IN TOROIDAL SYSTEMS

In Section 6, concerning systems with axially-flowing plasmas, it was shown that the conditions for equilibrium can indeed be fulfilled, but only by invoking certain active artificial aids. This is unsatisfactory, and prompts an inquiry as to whether, in some slightly more general 3-dimensional systems, these conditions could be fulfilled naturally.

The clue to the answer lies in a remark made in Section 4, to the effect that it is inconsistent to assume that the x-component of the line current in the boundary layer vanishes, while the y-component does not. It was this assumption that led Parker and Lerche to suppose that the ionic part J_{xi} of J_x must be closed by an electronic return current J_{xf}, and hence to conclude that equilibrium is impossible.

In Section 6, we concentrated on this problem of the closure of J_{xi}, together with the related problem of the conditions for B_y in the limits. We showed that, in a system in which a magnetic field surrounds a jet of flowing plasma, the current $I = \pi\ d\ J_{xi}$ can either close backwards, against the flow, remaining entirely within the plasma, or close forwards, along the jet in the direction of the flow, and return to the source through an external circuit. In the first case equilibrium is impossible, as Parker and Lerche had found, while in the second it is possible.

But why does a similar problem not arise in connection with the closure of J_{yi} ? Because, in systems with axial flow, this component of current closes naturally ; so does J_{yf}, and likewise the magnetic field component B_y. In any plane perpendicular to the axis of rotational symmetry, the cross-section of the magnetopause is a hollow area bounded by closed curves with no singularities : specifically, the curves are circles and the area is an annulus. The current J_{yi} flows around the annulus uniformly, without changing direction, and joins back onto itself. The lines of this current are closed loops, not needing to be completed by paths through external conductors.

The question therefore is whether J_{xi} can be made to close in the same straightforward way, so allowing equilibrium to exist naturally. The condition for this mode of closure is that lines drawn on the magnetopause, following the local direction of plasma flow, should also form closed loops. Clearly this is not possible with strictly axial flow, so we now turn our attention to systems with only approximately axial flow. On the scale of the diameter d of the magnetopause, such systems resemble those discussed in Section 6, but on a much larger scale the mean direction of flow is allowed to vary. Because the system is axial on the smaller scale, the conditions for natural closure of J_{yi}, J_{yf}, and B_y remain effective. But, in addition, the conditions for natural closure of J_{xi} will be created if the streamlines of the flow form closed loops on the larger scale. Thus the simplest system in which equilibrium can exist naturally is one in which the magnetopause has the shape of a torus (Fig. 9), the magnetic field being on the outside and the plasma on the inside.

How oould the existence of equilibrium be demonstrated experimentally ? Unfortunately the least elaborate system of this type, in which the plasma would occupy a volume around the minor axis of a toroidal solenoid (Fig. 10), would be useless in this respect because it would be grossly unstable. There is nothing in it to counteract two forces, namely the centrifugal force and the major-radial gradient of the magnetic field strength, which combine to push the plasma outwards. A further objection - if such were needed - is that there is nothing to keep the trapped electrons in the boundary layer stationary. Collisions with the free electrons and ions would soon get them flowing at the same speed as the rest of the plasma. Hence some slightly more sophisticated system must be envisaged.

The simplest system that has any chance of meeting both of the above requirements, for macroscopic stability and for restraint of the trapped electrons, is illustrated in Fig. 11. The structure of the magnetic field in this system may be pictured by imagining that, to the arrangement shown in Fig. 8, many more current-carrying rings are added, spaced uniformly along the axis of the system with a separation of the order of their radius, and that the axis is then bent around to form a circle. The currents in adjacent rings flow in opposite directions.

The resultant field consists of a set of ring cusps joined together to form a torus ; in the language of fusion plasma physicists, this field structure is a "toroidal picket-fence" (Tuck, 1963).

In the system of Fig. 11, the plasma flow can be considered to be approximately axial so long as the major diameter D of the torus is sufficiently large. The requirement is that the pressure difference across a minor diameter d, caused jointly by the major-radial gradient of the magnetic field strength and by the centrifugal force, be small compared with the mean kinetic pressure of the plasma. By the term "approximately axial" we mean that these forces, which are both due to the curvature of the toroidal minor axis, can be neglected when studying the flow on the scale of the minor diameter.

This system should be free from gross magnetohydrodynamic instabilities such as those to which the system of Fig. 10 is subject, because the confining field is everywhere convex towards the plasma (Rosenbluth and Longmire, 1957). Moreover, the trapped electrons in the boundary layer are kept stationary with respect to the magnetic field by the fact that the field lines emerge from the layer at the cusps, just as they do in the Earth's magnetopause (Fig. 1).

It is less clear, however, that the conditions for equilibrium of the boundary layer between the flowing plasma and the magnetic field will exist naturally everywhere in such a system. Streamlines of the plasma flow form closed loops as required, but on the other hand the magnetic field lines that thread the boundary layer are not continuous around the torus, and this circumstance may affect the mode of closure of J_{xi}. In the toroidal system of Fig. 11, as in the linear one of Fig.8, J_{xi} is not constant along the jet of flowing plasma. It is greatest where the magnetic field at the surface of the jet is strongest, i.e. in the mid-plane of each of the current-carrying rings. As in Section 6, the question is how this maximum value of J_{xi} closes : forwards around the entire torus, or backwards through the plane of the ring, or in both ways at once ? This question can be re-phrased in terms of J_x, which is necessarily constant around the torus : is J_x equal to this maximum value of J_{xi}, or is it zero, or has it some intermediate value ?

Our opinion is that J_x is positive, though somewhat less than the maximum value. This view is based on a consideration of the effects of electron-ion collisions. To begin with, let us consider only collisions in the main body of the plasma, neglecting those in the boundary layer. In the steady state and on the spatial average, there can be no electric field in the plasma parallel to the toroidal minor axis, so the frictional force due to electron-ion collisions, however weak, ensures that the spatial average flow speeds of the electrons and of the ions are the same. This means that there is a toroidal current, since there are fewer flowing electrons than there are ions, the difference being the number of electrons trapped on the magnetic field lines in the boundary layer. On this basis, J_x would be equal to the spatial average of J_{xi}. Now let us also consider the collisions between the free particles and the trapped electrons. Clearly they will decelerate the free electrons more effectively than the much heavier free ions. Hence, provided that whatever mechanism is used to maintain the plasma flow does not have a contrary effect, J_x should be somewhat greater than the spatial average of J_{xi}, though less than the maximum value of this quantity ; there is already some experimental evidence in support of this view (see Section 8).

Though the existence of a large positive J_x favours equilibrium in a general way, the fair chance that J_{xi} exceeds J_x in certain regions of the magnetopause, centred on the mid-planes of the current-carrying rigns, implies that J_{xf} becomes negative there. As was explained near the end of Section 6, this state of affairs may compromise equilibrium. Hence the question of whether equilibrium exists everywhere on the magnetopause, in a system of the type shown in Fig. 11, must remain open until the conditions for it have been derived quantitatively, with the ratio J_{xf}/ J_{xi} as a parameter in addition to U/V_i.

In conclusion, if one seeks to demonstrate the existence of equilibrium experimentally, the system of Fig. 11 appears to hold out the best possibility, though falling short of certainty. Further study is needed to clarify this point. Assuming provisionally that the outcome of such a study will be favourable, we now inquire about the eventual applications of this type of system.

8. EVENTUAL APPLICATION TO CONTROLLED FUSION

In Section 7, it was argued that the boundary layer between a flowing plasma and a magnetic field would exist in equilibrium naturally in toroidal geometry. Obviously this result is irrelevant to the physics of planetary magnetospheres. However, the fact that the torus is one of the most popular shapes for magnetically-confined plasmas in research on controlled fusion suggests that it might have some relevance to fusion.

This question is particularly intriguing because one of the main aims of fusion research is to use the confining magnetic fields efficently. The usual measure of efficiency is the parameter β, which is the ratio of the plasma-kinetic pressure to the magnetic pressure. Systems of the type considered here, in which the magnetic field is totally expelled from the main body of the plasma, would be particularly efficient in this respect.

Several previous proposals have been made for the confinement of flowing plasmas by means of magnetic fields having this structure. There is notably the "Polytron", which is illustrated in Fig. 12 (Haines, 1977). This is a low-β device, however : the magnetic field penetrates the plasma fully, almost all the electrons are trapped, and only the ions flow around the torus. As regards β, the present proposal has more in common with the "moving toroidal picket-fence" (Fig. 13), in which, rather than have the plasma flow through a stationary cusp-shaped magnetic field, the plasma is stationary while the entire pattern of cusps is made to move around the torus (Tuck, 1963) ; thus, with respect to the field, the plasma can still be considered to be flowing. This result is achieved by using a larger number of rings to produce the field, by exciting them with radio-frequency currents, and by arranging for there to be appropriate phase differences between the currents in adjacent rings. Devices of this type have been built by various workers, most recently by Osovets and Popov (1976). Unfortunately, they all suffer from the limitation that r.f. fields sufficiently strong to confine a fusion plasma cannot be produced without a prohibitive outlay of power, at any rate using current technology.

In contrast with these previous concepts, the present proposal combines high β with the use of a stationary confining magnetic field (see Fig. 11). The proposed device, in which the plasma as a whole would flow along the minor axis of a stationary toroidal picket-fence, has been named "plasma storage ring" in an earlier report (Storey, 1977).

In all such systems, the reason for having the plasma in a state of approximately axial flow is to improve the confinement. If the plasma were stationary, it would leak out of the system through the cusps, at a rate which is inacceptable for the application to fusion (Spalding, 1971 ; Haines, 1977). But if the plasma is flowing supersonically, most of the ions jump over the cusps, taking the free electrons with them. Only a small fraction, for which the axial component of the random thermal velocity is comparable and opposite to the bulk flow velocity, is able to escape. On these grounds, the flow may be expected to reduce the rate of leakage by a multiplicative factor of the order of exp $(- U^2/V_i^{\,2})$; at a guess, a flow speed $U > 10\ V_i$ might be needed to achieve satisfactory confinement.

The experiments of Osovets and Popov (1976) with a moving toroidal picket-fence throw some light on the question as to whether, on the magnetopause of a plasma storage ring, microscopic equilibrium could exist everywhere. The following is a quotation from their paper : "A fundamental feature of a discharge in a traveling magnetic field is the excitation, in the wave-propagation direction, of a strong, quasi-constant drift current ; if the initial pressure of the working gas is low, and if the peak r.f. field is high, the magnetic field produced by this current becomes comparable to the r.f. field". The current referred to is due to the trapped electrons, which move with the moving field lines. In a frame of reference that also moved with the field lines, this current would be regarded as being due to the excess of free ions over free electrons, moving in the opposite direction, and corresponding to a large positive J_x. As explained in Sections 6 and 7, the existence of such a current is a necessary - though perhaps not sufficient - condition for microscopic equilibrium to exist everywhere on the magnetopause.

In order to maintain macroscopic equilibrium in these experiments, it was necessary to cancel the outward force due to the major-radial gradient of the magnetic field of the toroidal current. For this purpose, a transverse magnetic field was applied to the plasma, in a direction parallel to the major axis. The interaction of this field with the toroidal current gave rise to a force directed inwards, towards the axis. The same arrangement was adopted in experiments on the Polytron, mainly to cancel the centrifugal force (Kilkenny et al., 1973). Probably it would also be needed in any experimental study of the proposed plasma storage ring, where it would help to maintain microscopic as well as macroscopic equilibrium (Storey, 1977).

Having discussed macroscopic equilibrium and stability, and also microscopic equilibrium, there remains the question of microscopic stability. The currents flowing in the boundary layer may excite micro-instabilities which disrupt it. A study of stability normally proceeds in two successive phases : (1) calculation of the equilibrium plasma structure ; (2) examination of the stability of this structure with respect to all plausible perturbations. In the present paper, we have taken the preliminary step of proving that the boundary layer can exist in equilibrium, so opening the way to the study of its structure and thence of its stability. These problems, which can be treated in the 1-dimensional approximation, will form the subjects of later papers.

On the question of microscopic stability, the experimental results of Osovets and Popov (1976) are inconclusive. They claim to have achieved stable confinement, but their data suggest that the r.f. magnetic field penetrated further into the plasma than it would have done had the boundary layer been perfectly stable (see their Fig. 5). In these experiments, the plasma densities were of the order commonly envisaged for fusion, around 10^{14} cm^{-3}, but the ion temperatures were much lower, no more than a few tens of electron-volts.

Of course, in order to achieve controlled fusion, it is not enough to be able to confine a plasma stably at the proper density and temperature. At the very least, one much also be able to get the plasma into this state and to keep it there. Some suggestions as to how these objectives might be attained in systems of the type illustrated in Fig. 11 have been made in an earlier report (Storey, 1977).

Moreover, systems of this type pose the peculiar problem of how to create and to maintain the plasma flow. Means for doing this exist for the Polytron and for the moving toroidal picket-fence, but neither is applicable to the plasma storage ring. The problem is how to exert a bulk force on a magnetically-confined but field-free plasma, in a direction tangential to the magnetopause. Some solutions have been proposed in the report cited above. Here it suffices to say that they are related to the hypothetical "viscous interaction" between the solar wind and the magnetosphere, by which means momentum may be transferred from the one to the other without there being necessarily any transfer of matter (Axford, 1964, 1969).

In sum, plasma storage rings may conceivably have applications to controlled fusion. They would offer the advantage of confining the plasma at a high value of β, provided that all the other difficulties mentioned above can be overcome. As usual, the basic problem is that of the stability of the confinement ; a continuation of the work begun in this paper would help to solve it.

9. CONCLUSIONS

We now recapitulate the main conclusions from each section of this paper, except for Sections 1-3 which are introductory.

SECTION 4

The case presented by E.N. Parker and I. Lerche, against the possibility of a boundary layer existing in small-scale equilibrium when the plasma is flowing supersonically, contains certain implicit assumptions about the closure of the current in the layer in the direction of plasma flow, and also about the conditions in the limits for the secondary magnetic field that this current creates. These assumptions are not necessarily valid. Others, which are equally acceptable physically, imply that equilibrium is possible.

SECTION 5

The factor that decides which set of assumptions holds good in a given case is the large-scale 3-dimensional structure of the system. In this way, the macroscopic structure governs the conditions for microscopic equilibrium.

SECTION 6

In systems with strictly axial flow, equilibrium cannot exist at supersonic flow speeds if the large-scale structure is such that the plasma confines the magnetic field, but it can if the field confines the plasma. In the latter case, however, equilibrium can be secured only by making the axial current in the jet of flowing plasma close artificially through an external circuit.

SECTION 7

In systems with only approximately axial flow, and in which the field confines the plasma, equilibrium would occur naturally if the axis of the jet were bent around into a circle, so that the magnetopause had the form of a torus. However, the need for large-scale (magnetohydrodynamic) stability leads to the adoption, for the confining magnetic field, of a toroidal picket-fence structure involving a succession of cusps. It is not certain whether, in such a system, the conditions for small-scale equilibrium can be satisfied everywhere on the magnetopause.

SECTION 8

If equilibrium did indeed exist everywhere, this system might be useful for the high-β confinement of fusion plasmas. In this application, the speed of flow would have to be an order of magnitude greater than the ion thermal speed, so as to impede the plasma from escaping through the cusps. There is a serious doubt as to whether, at such speeds, the magnetopause could be microscopically stable. Only after satisfaction has been obtained on this fundamental point will it be worth while to study the other problems, such as how to create and to maintain the plasma at the requisite density, temperature, and flow speed.

The outstanding conclusion of the present study, namely that the boundary layer between a magnetic field and a supersonically-flowing plasma can exist in small-scale equilibrium, clears the path towards the development of a quantitative kinetic theory of this type of magnetopause. The main points that need investigation are the exact conditions for equilibrium, the structure of the corresponding boundary layer, and its microscopic stability. All these properties of the layer can be studied in the 1-dimensional approximation. They are discussed further in the companion Paper II, which outlines a programme for their future study.

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FIGURE CAPTIONS

1. Section of the Earth's magnetosphere, in the plane of the solar-wind velocity vector and the magnetic dipole (after Fig. 11 of Rosenbauer et al., 1975).

2. Idealized 1-dimensional magnetopause between an isotropic plasma and a magnetic field that is constant in direction.

3. Trajectories of plasma ions and electrons incident normally on the magnetopause, when the polarization electric field is neutralized completely by trapped cold electrons (after Fig. 4 of Willis, 1975).

4. Approximate profiles of the free-electron density N_f and of the ion density N_i, normalized with respect to the density N_o of the uniform plasma at $z = -\infty$, as functions of the position coordinate z across the magnetopause, when the plasma is stationary.

5. Illustrating how the variation of the x-component of the magnetic field across the magnetopause can be considered as resulting from the summation of a uniform externally-applied primary field, and a secondary field due to the magnetopause current sheet : (A) situation in the plane containing the applied field and the normal to the magnetopause ; (B) situation in the plane perpendicular to the applied field.

6. System in which a collisionless isotropic plasma flows around the field of a magnetic dipole oriented parallel to the general direction of flow.

7. Sections through two systems with axial flow, in a plane perpendicular to the axis : (A) system of Fig. 6, in which the plasma surrounds the magnetic field ; (B) system of Fig. 8, in which the field surrounds the plasma. The broken lines are the circuits around which one applies Ampère's law, to derive the conditions for B_y in the limits.

8. System in which a jet of collisionless isotropic plasma flows along the axis of the magnetic field produced by a current-carrying ring.

9. A torus : geometry and nomenclature.

10. Toroidal system for the confinement of a flowing plasma, in which microscopic equilibrium would exist naturally, but which would be macroscopically unstable.

11. Toroidal system for the confinement of a flowing plasma, which should ensure macroscopic stability, but in which microscopic equilibrium might not exist everywhere.

12. Illustrating the principle of the Polytron. The confining magnetic field is static, but the ions are accelerated by the changing flux of another field (not shown) which threads the torus, parallel to the major axis.

13. Illustrating the principle of the moving toroidal picket-fence. The confining magnetic field is produced by exciting the conducting rings with radio-frequency current (here shown as 3-phase).

THE

EARTH'S

MAGNETOSPHERE

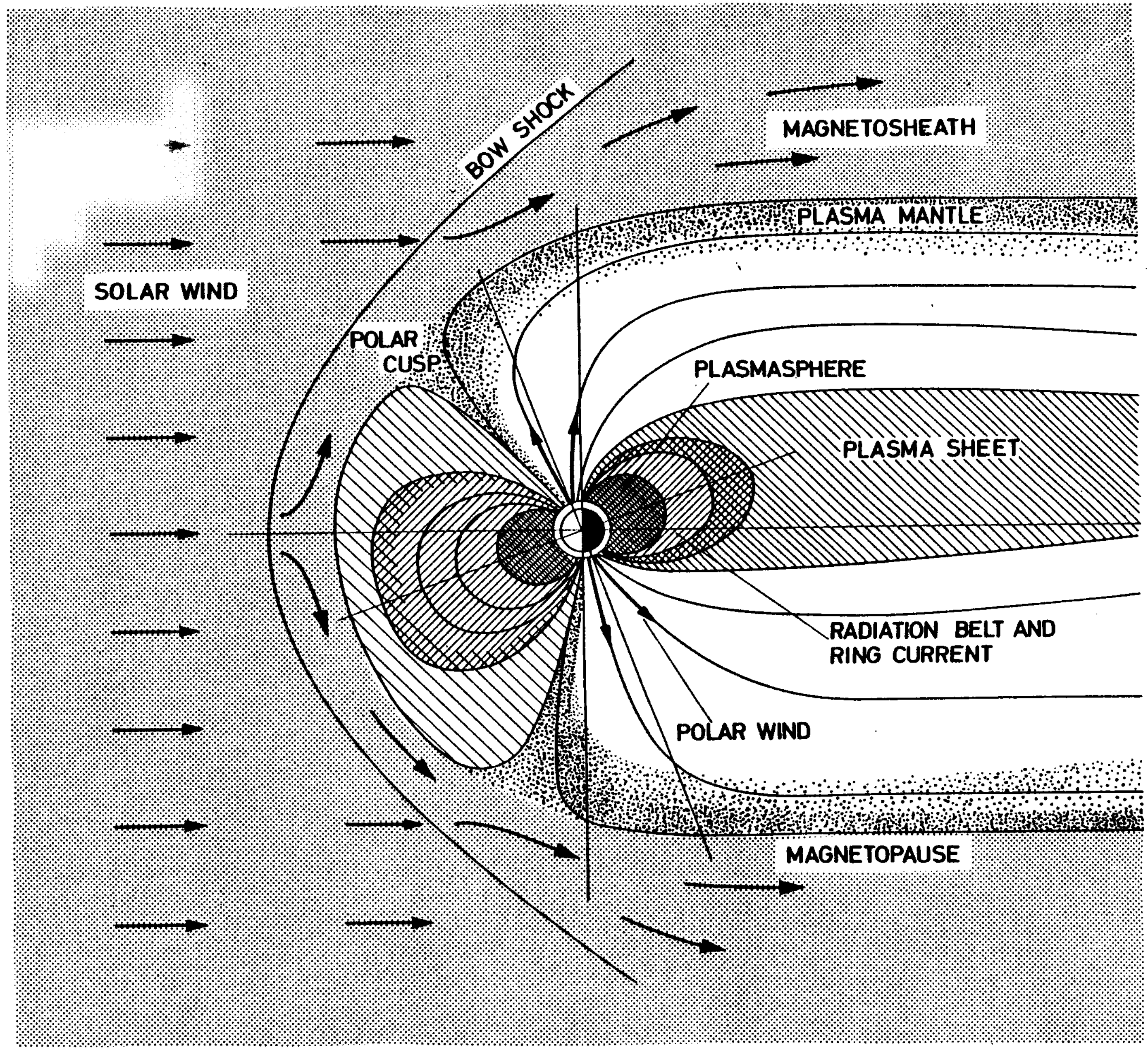


FIGURE 1

STATEMENT OF THE PROBLEM

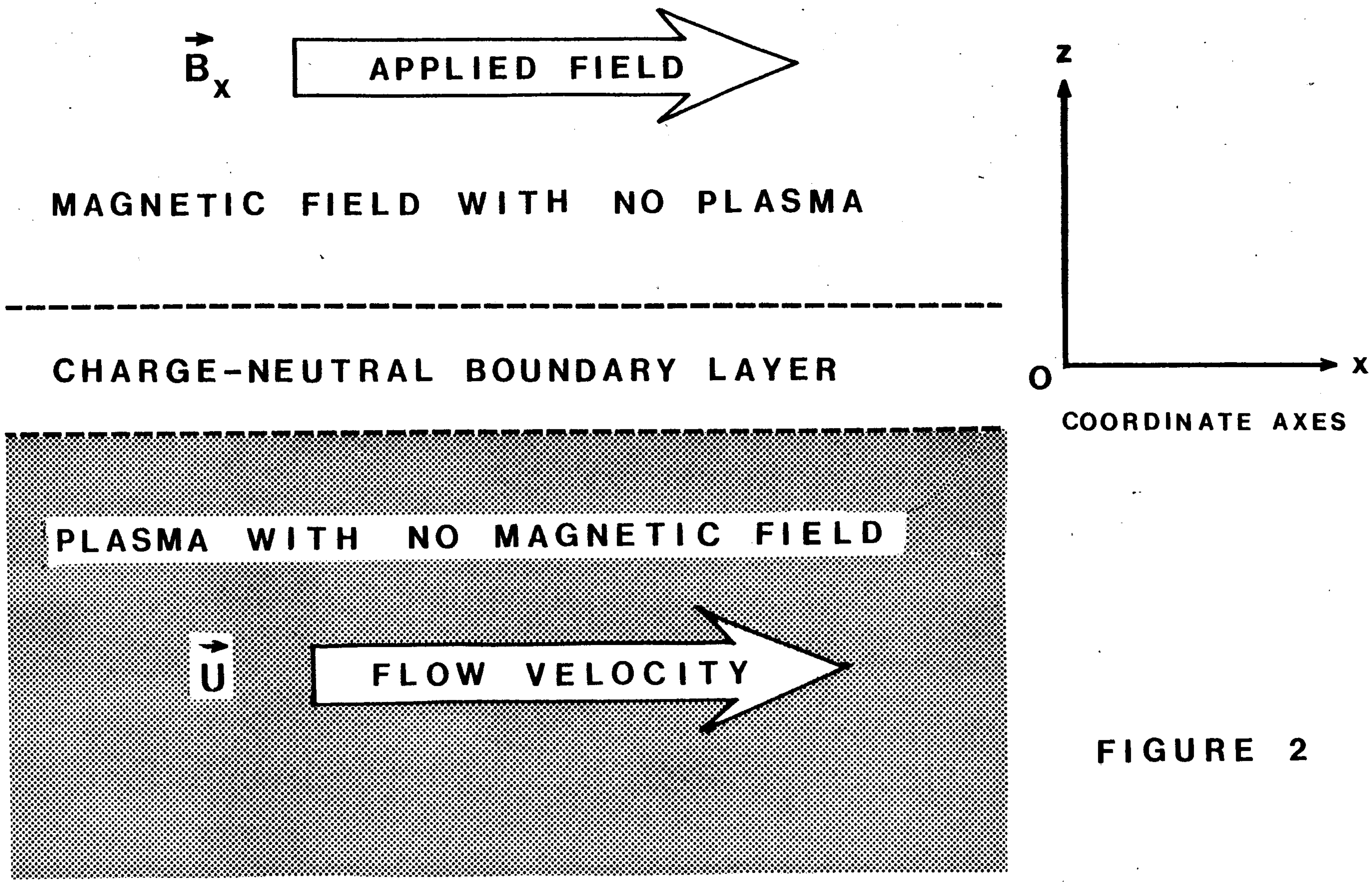


FIGURE 2

FREE PARTICLE TRAJECTORIES IN THE yz PLANE

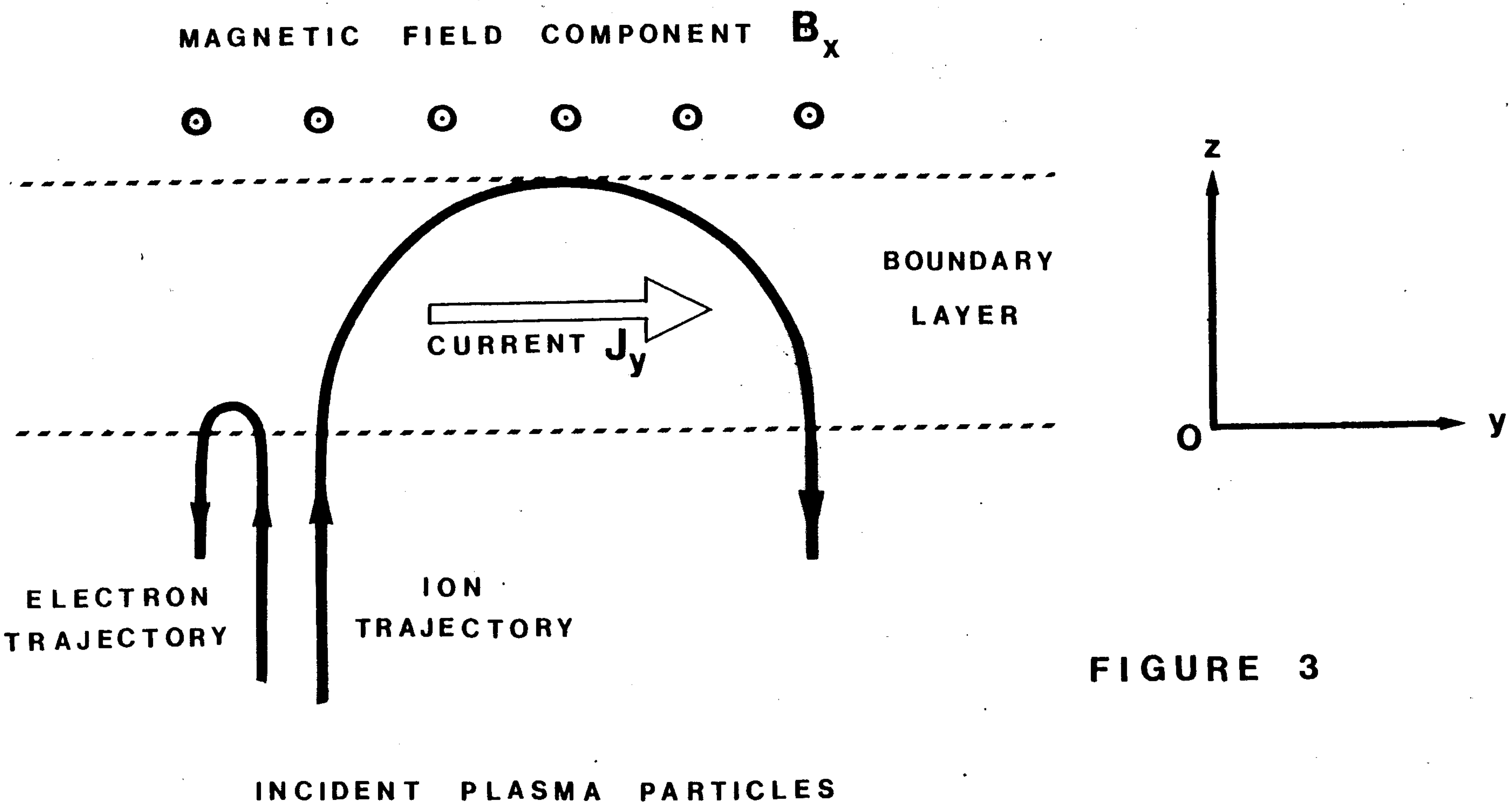


FIGURE 3

PARTICLE DENSITY PROFILES, WITH PLASMA STATIONARY

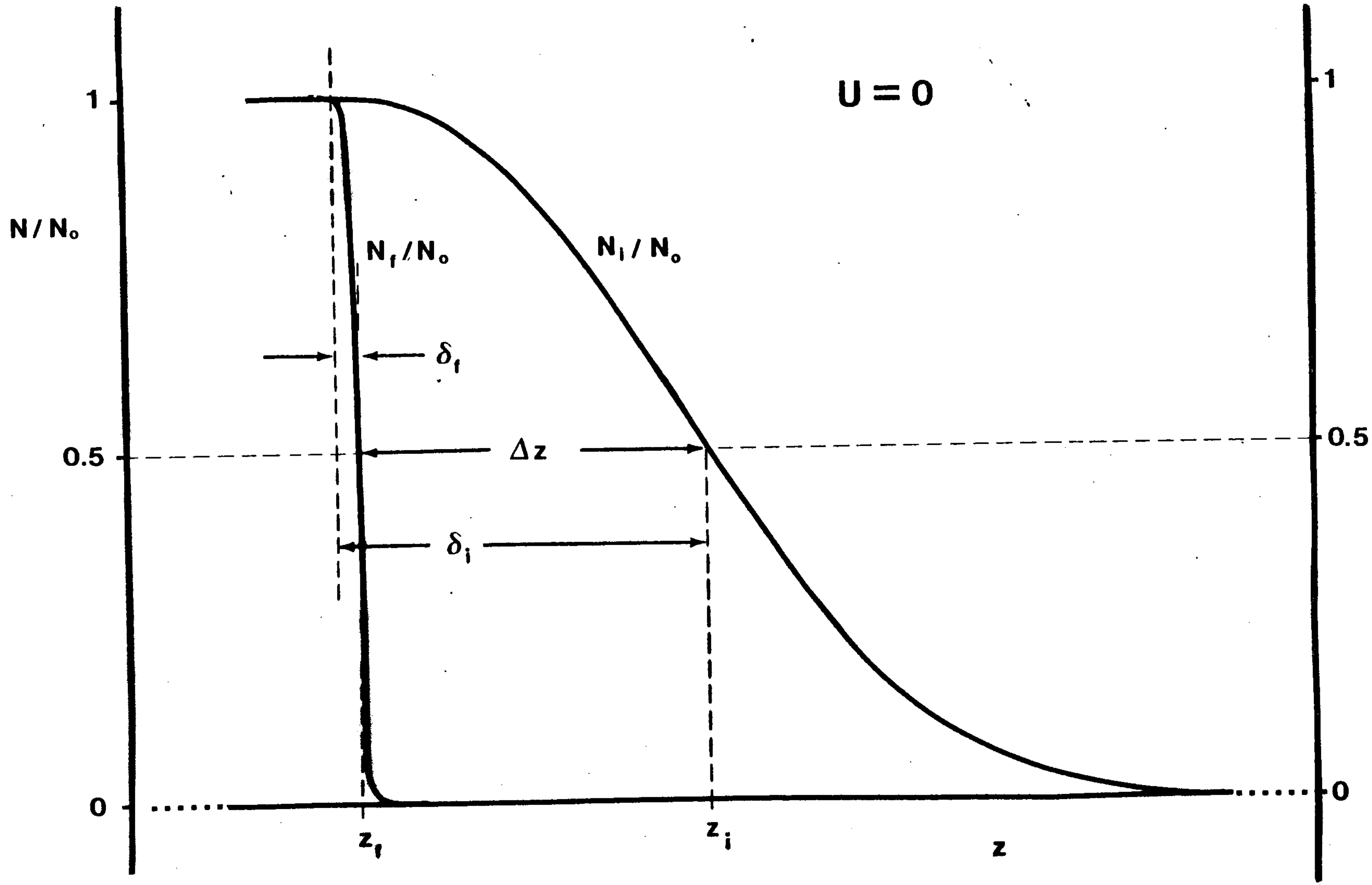
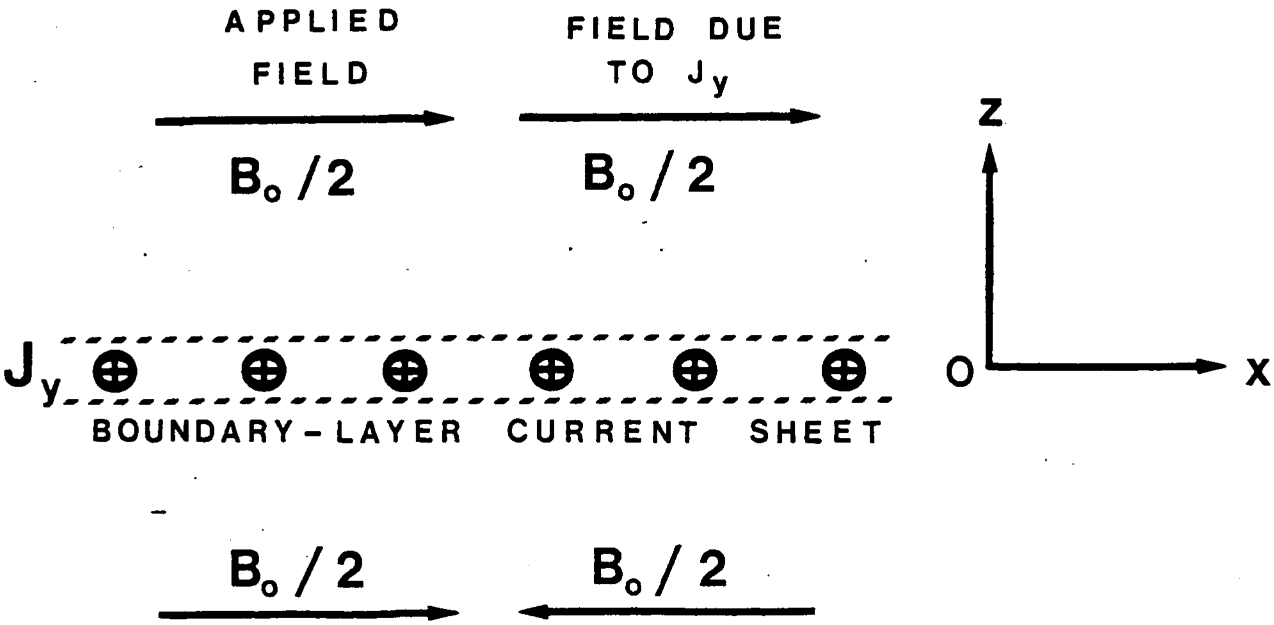


FIGURE 4

MAGNETIC FIELDS IN THE xz PLANE

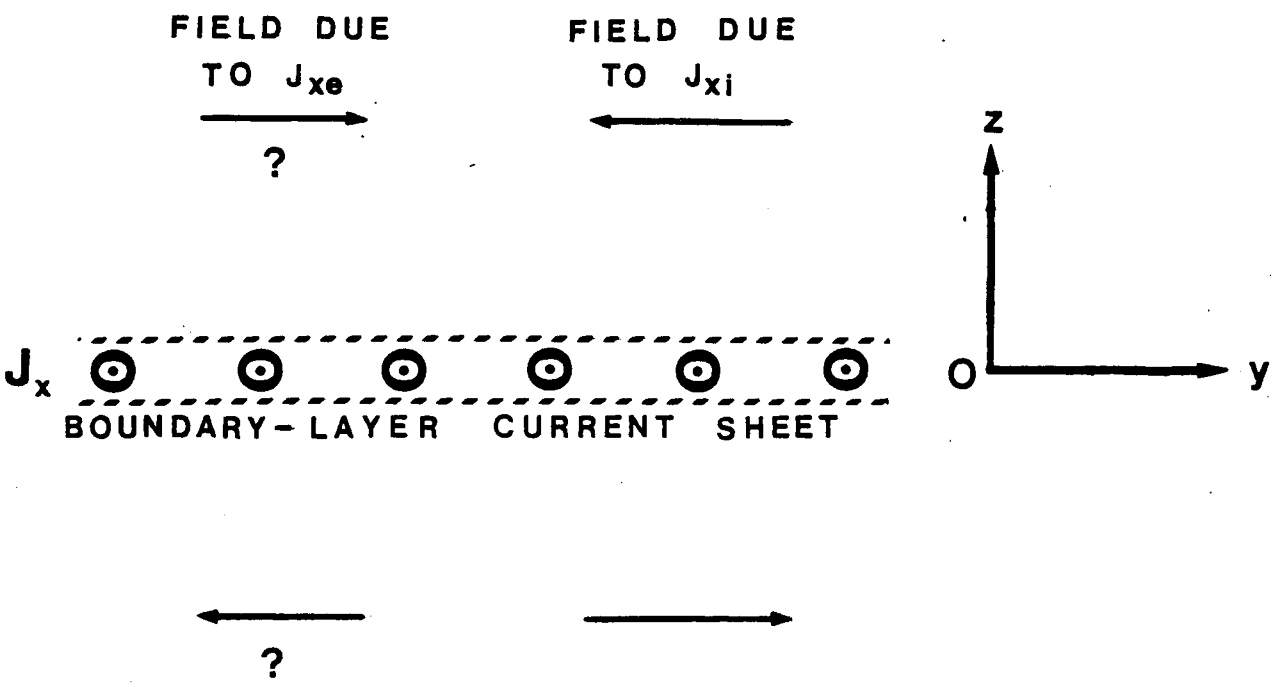


LINE CURRENT DENSITY $\quad J_y = J_{yi} + J_{ye}$

$$J_{yi} \simeq J_{ye}$$

FIGURE 5A

MAGNETIC FIELDS IN THE yz PLANE



LINE CURRENT DENSITY $J_x = J_{xi} + J_{xe}$

$$J_{xi} \simeq \left(\frac{U}{V_i}\right) J_{yi}$$

FIGURE 5B

FLOWING PLASMA ENCLOSING A MAGNETIC FIELD

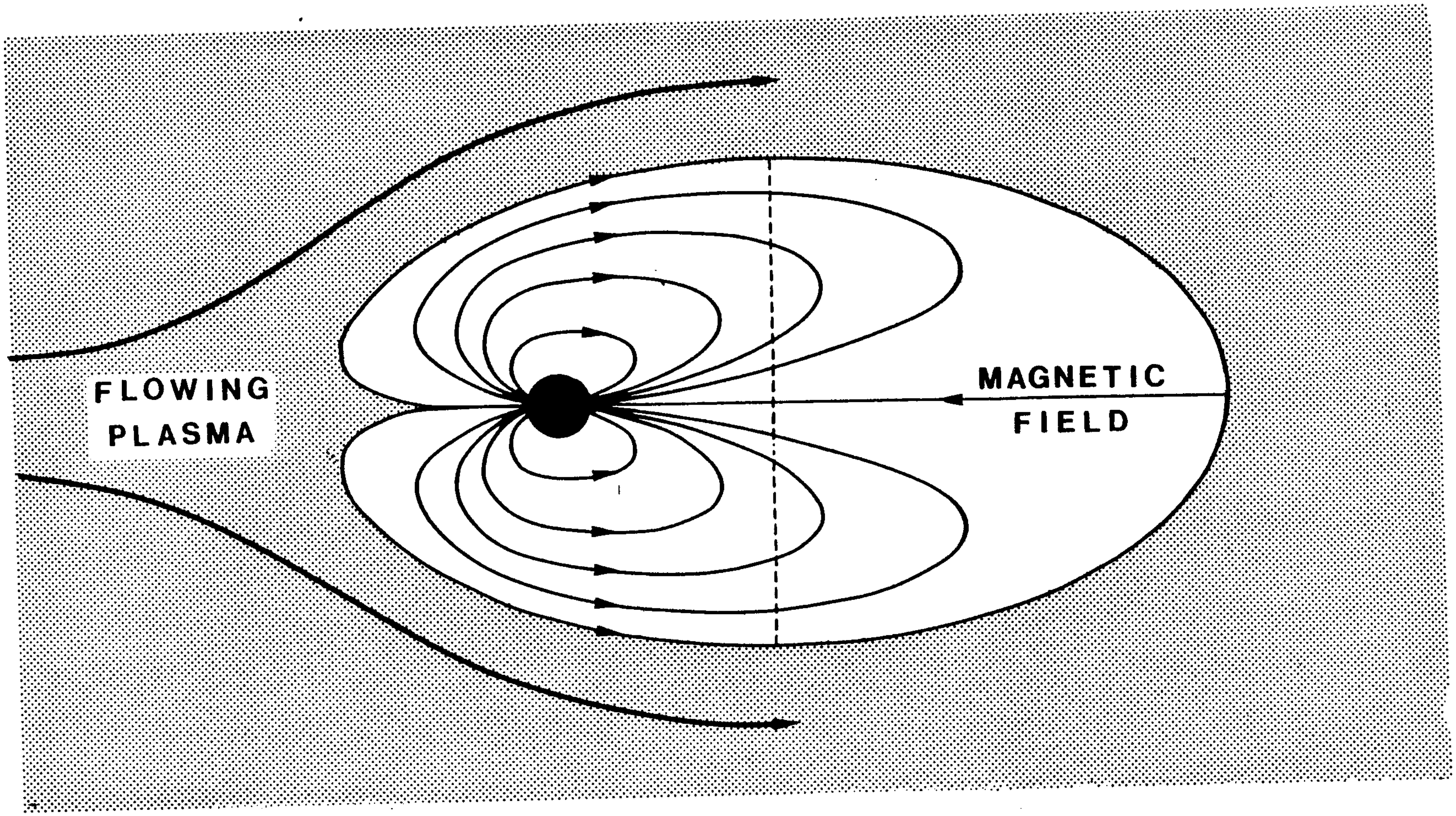


FIGURE 6

CIRCUITS FOR APPLYING AMPÈRE'S LAW

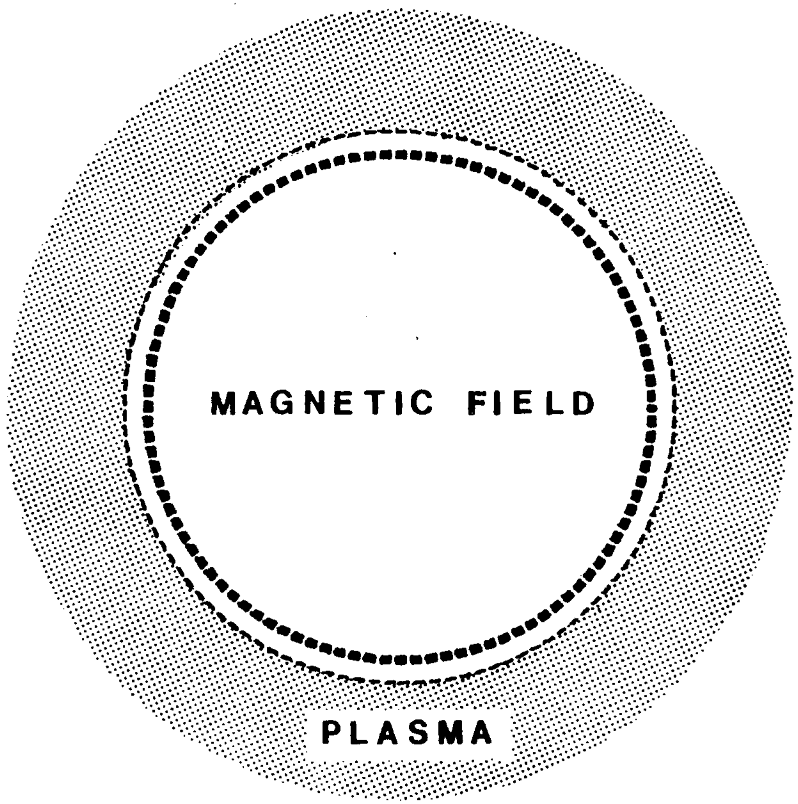


FIGURE 7A

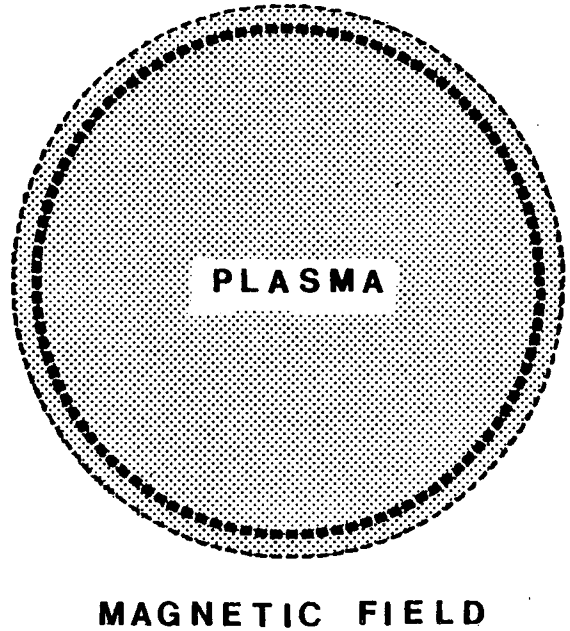


FIGURE 7B

MAGNETIC FIELD ENCLOSING A FLOWING PLASMA

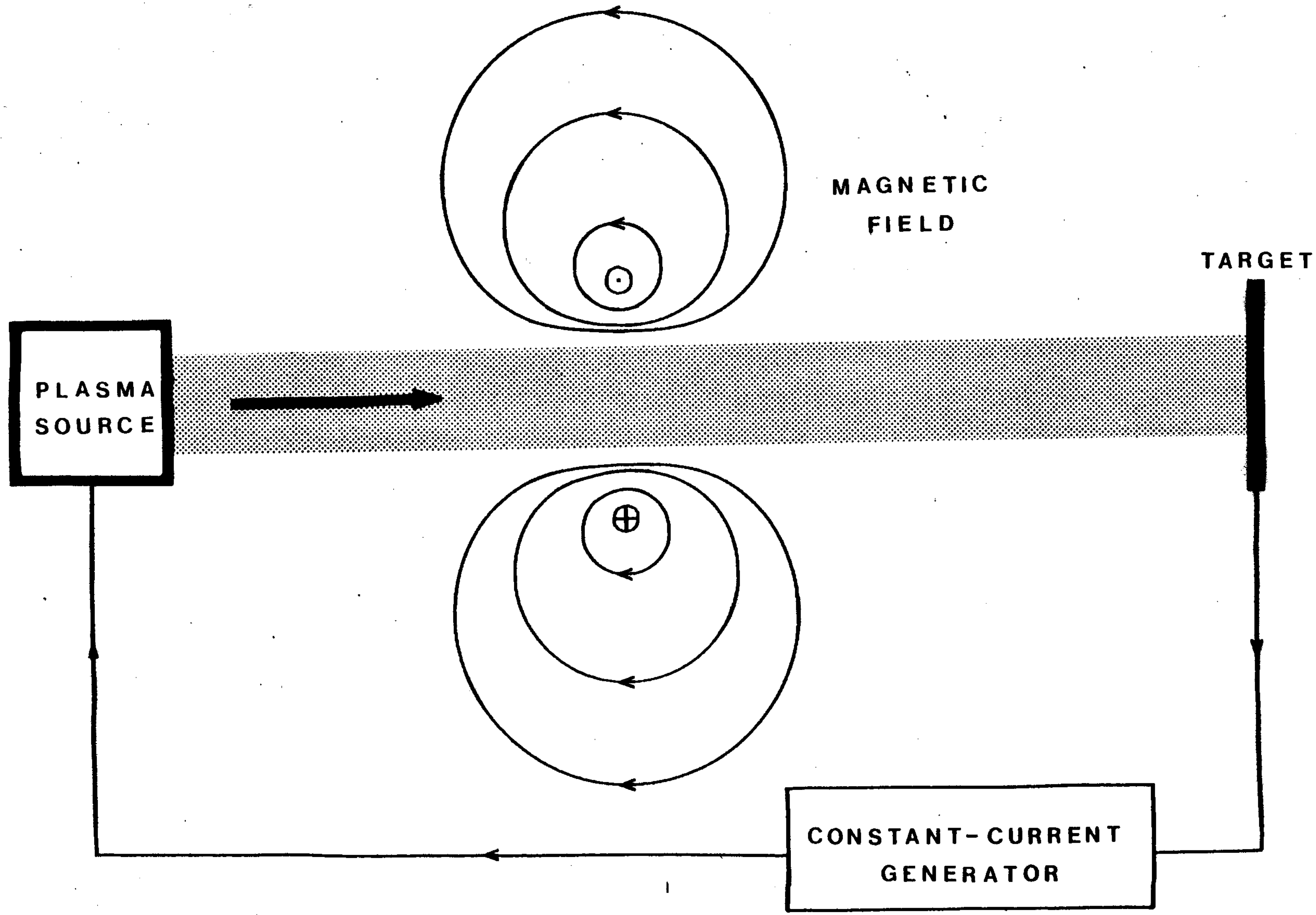


FIGURE 8

A TORUS : GEOMETRY AND NOMENCLATURE

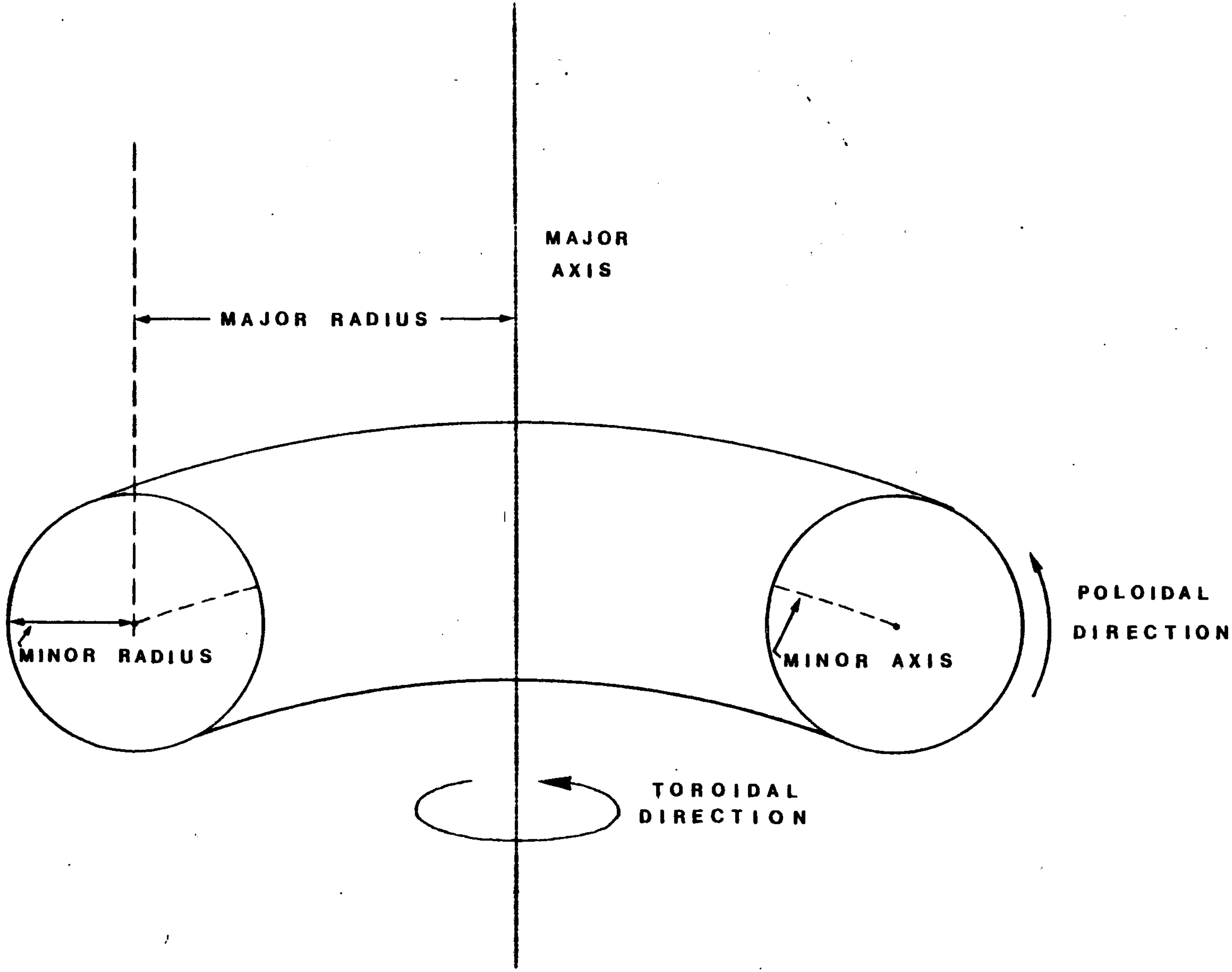


FIGURE 9

TORODIAL SOLENOID

PLASMA STORAGE RING

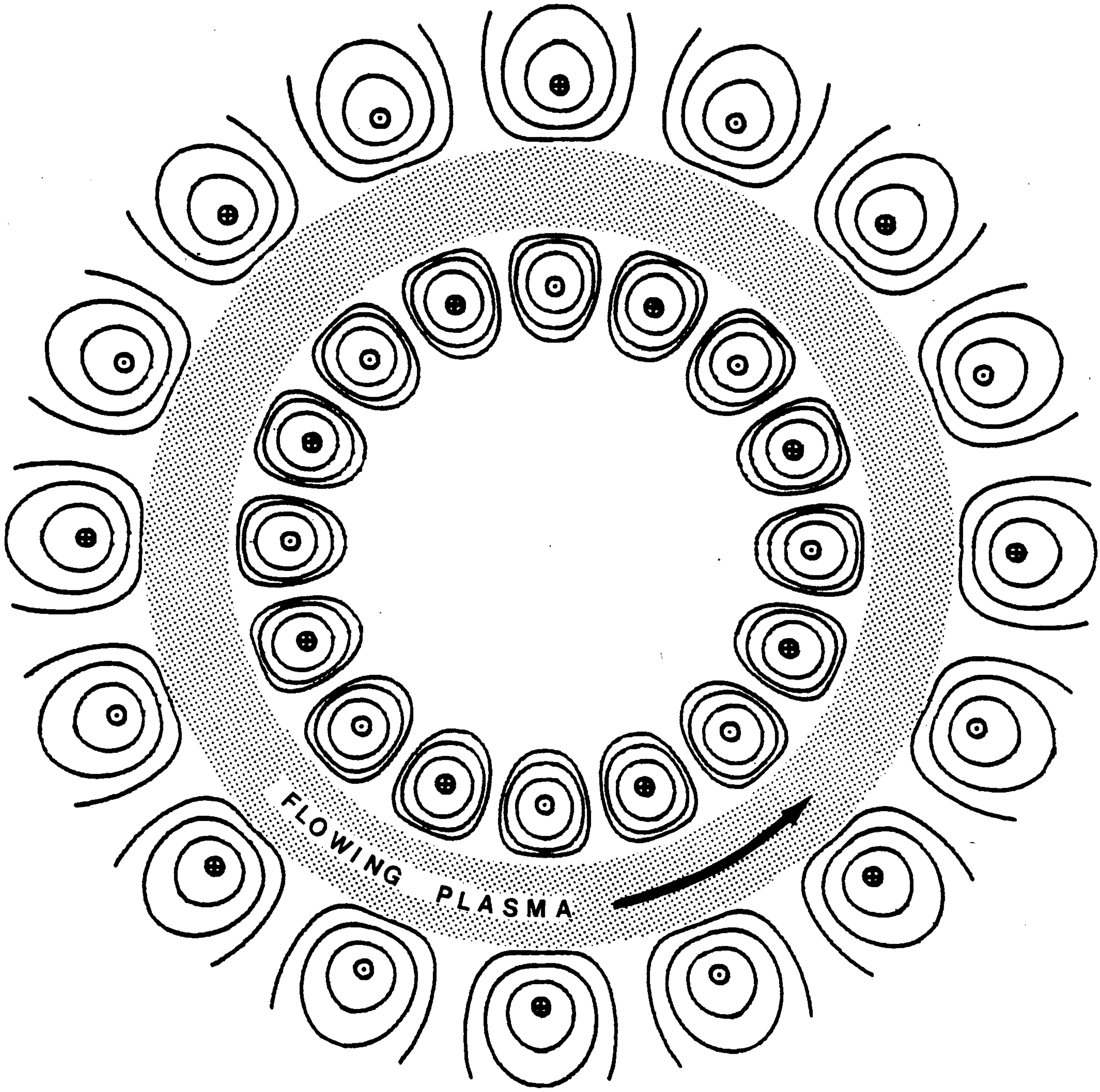


FIGURE 11

POLYTRON

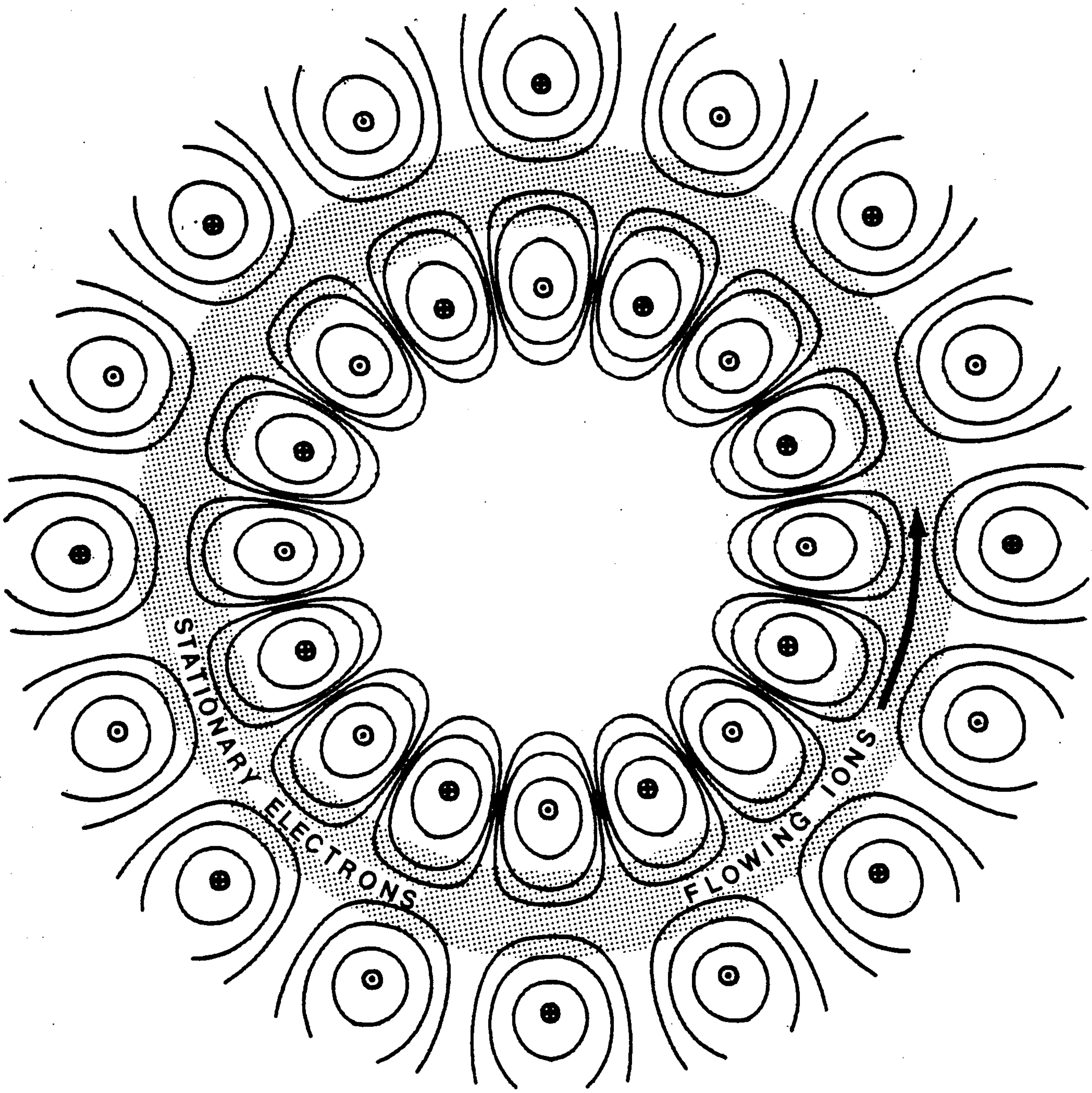


FIGURE 12

MOVING PICKET - FENCE

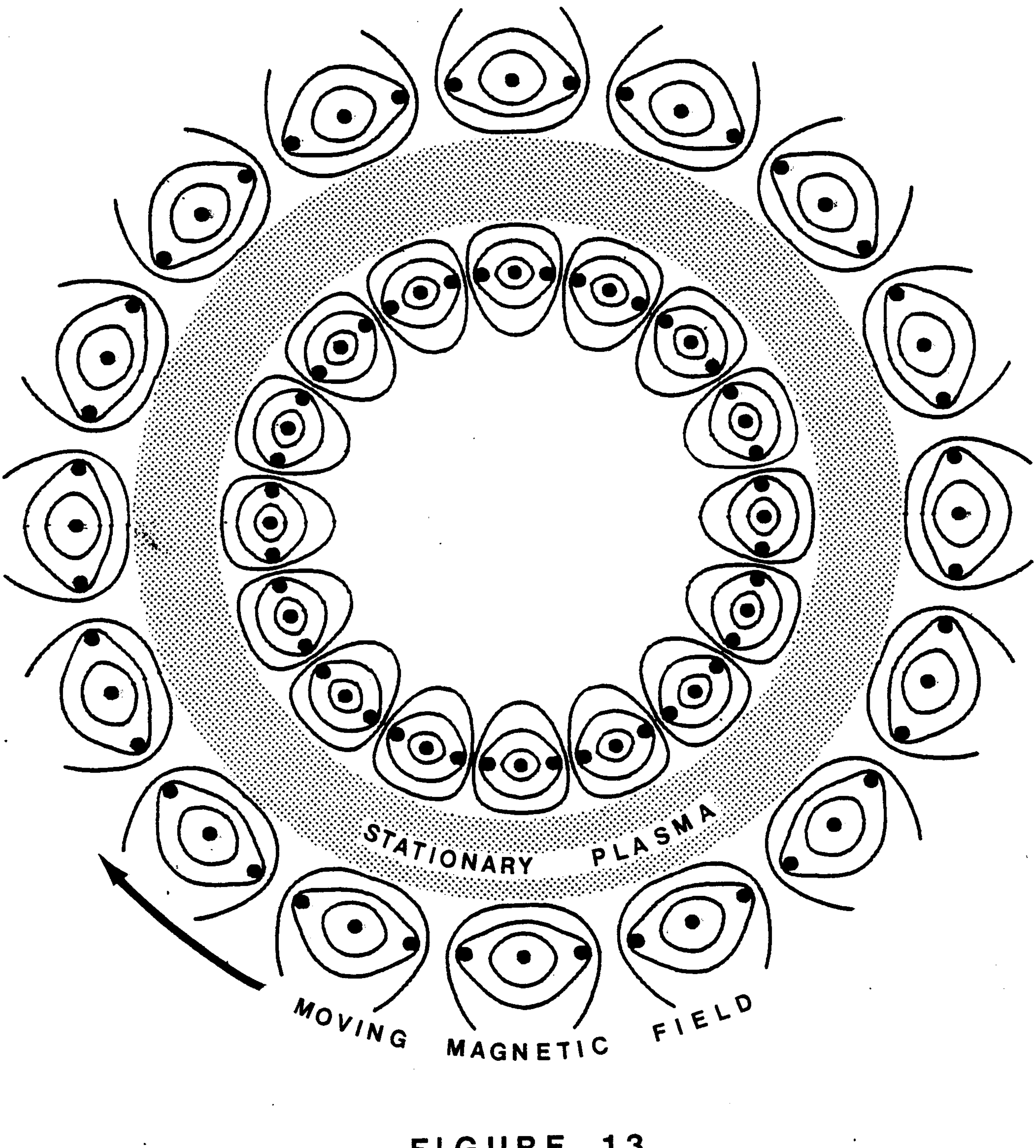


FIGURE 13

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