HAL open science

The analysis of 6-component measurements of a random electromagnetic wave field in a magnetoplasma. I: The direct problem

L.R.O. Storey, François Lefeuvre

▶ To cite this version:

L.R.O. Storey, François Lefeuvre. The analysis of 6-component measurements of a random electromagnetic wave field in a magnetoplasma. I: The direct problem. [Research Report] Note technique CRPE n° 12, Centre de recherches en physique de l'environnement terrestre et planétaire (CRPE). 1975, 36 p. hal-02191369

HAL Id: hal-02191369

https://hal-lara.archives-ouvertes.fr/hal-02191369

Submitted on 23 Jul 2019

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

RP 182 (5)

CENTRE NATIONAL D'ETUDES
DES TELECOMMUNICATIONS

CENTRE NATIONAL DE LA
RECHERCHE SCIENTIFIQUE

CENTRE DE
RECHERCHES
EN PHYSIQUE DE
L'ENVIRONNEMENT
TERRESTRE
ET PLANETAIRE

CRPE

NOTE TECHNIQUE
CRPE / 12

The analysis of 6 - component measurements of a random electromagnetic wave field in a magnetoplasma.

I - THE DIRECT PROBLEM

by

L.R.O. STOREY
F. LEFEUVRE

BIBLIOTHEQUE DU CENTRE DE DOCUMENTATION C.N.R.S.

14 DEC. 1976

CENTRE NATIONAL D'ETUDES
DES TELECOMMUNICATIONS

CENTRE NATIONAL DE
LA RECHERCHE SCIENTIFIQUE

CENTRE DE RECHERCHE EN PHYSIQUE DE
L'ENVIRONNEMENT TERRESTRE ET PLANETAIRE

NOTE TECHNIQUE CRPE/12

THE ANALYSIS OF 6-COMPONENT MEASUREMENTS OF A RANDOM ELECTROMAGNETIC WAVE FIELD IN A MAGNETOPLASMA. I - THE DIRECT PROBLEM

by

L. R. O. STOREY and F. LEFEUVRE

CRPE/PCE

45045 ORLEANS-LA-SOURCE, FRANCE

OCTOBRE, 1975

LE DIRECTEUR

THE ANALYSIS OF 6-COMPONENT MEASUREMENTS OF A RANDOM ELECTROMAGNETIC WAVE FIELD IN A MAGNETOPLASMA.

I - THE DIRECT PROBLEM

by

L.R.O. STOREY and F. LEFEUVRE

ABSTRACT

This is the first of a series of papers, the general subject of which is how to interpret a set of simultaneous measurements of the three electric and three magnetic components of a random electromagnetic wave field in a magnetoplasma. The point at which the measurements are made is assumed to be stationary with respect to the plasma. In this first paper, the following problems are treated : how to define, within the framework of classical electrodynamics, a distribution function that characterizes the statistics of a linear random electromagnetic wave field in a lossless magnetoplasma ; the direct problem of predicting the statistical properties of measurements of the six components of a field of this type, when the distribution function is known.

CONTENTS

I. GENERAL INTRODUCTION

This paper presents the first step in the development of a method for the analysis of experimental data on linear random (i.e. weakly turbulent) electromagnetic wave fields in a magnetoplasma. The method is suitable, in particular, for analysing measurements of the six components of the fields of certain natural very-low-frequency (VLF) or extremely-low-frequency (ELF) electromagnetic wave phenomena, in the study of the interactions between waves and energetic particles in the earth's magnetosphere.

The wave phenomena in question are those that appear to be continuous and structureless, such as VLF and ELF hiss, in contrast to discrete phenomena such as whistlers or VLF emissions which have clear-cut structures in the frequency-time plane (RUSSELL et al. , 1972). Their interactions comprise the plasma instabilities that produce them, and also the perturbations that they cause to particles other than those involved in their production.

In the development of the theory of magnetospheric wave-particle interactions during recent years, the situations envisaged have become progressively more and more complex. The earliest work concerned beams of mono-energetic particles with a single pitch angle, interacting with plane monochromatic waves propagated at a fixed angle to the magnetic field. Soon, however, this situation was seen to be unrealistic. In the magnetosphere, the interacting particles are spread in energy and in pitch angle, and must be characterized by their statistical distribution with respect to these two variables. Accordingly the next developments in the theory concerned interactions between the population of energetic electrons or protons, with a given distribution function, and plane monochromatic waves. In point of fact, this is a reasonable model for the interactions involving natural **whistlers** from lightning strokes. Hence much theoretical work has been done on interactions between whistlers and energetic particles, in particular on whistler -triggered V L F emissions, and on the similar discrete emissions that can be triggered by artificial signals of fixed frequency radiated from VLF transmitters on the ground.

Equally important, however, are the interactions involving natural VLF noise that is generated entirely within the magnetosphere, doubtless throughout large regions, and which covers a wide range of frequencies. At a given point of observation, which may be either inside or outside the source region, the corresponding waves arrive from different parts of this region by different paths, with different wave-normal directions, and all frequencies are present simultaneously. Accordingly, in some of the most recent work on the pitch-angle diffusion of radiation-belt particles under the influence of natural VLF waves, the latter are recognized as being spread in frequency and in direction of propagation, and hence -like the particles- as needing to be characterized by some kind of distribution function (LYONS et al. , 1972 ; SCHULZ and LANZEROTTI, 1974).

Therefore, in order to provide inputs to the theory and to test its outputs, one needs to be able to determine, from experimental data, distribution functions for the waves as well as for the particles. Methods for measuring particle distribution functions are well developed, yet almost no thought has been given as to how to make the corresponding measurements for waves ; indeed, only recently has the need for such measurements been acknowledged clearly (STOREY, 1971).

The present paper is an initial step towards the development of one possible method for measuring distribution functions for electromagnetic waves in space plasmas : electrostatic waves require different methods, which will not be discussed here.

The first question that arises is the choise of the data on which to base the method, and the answer depends on what practical limitations to data collection are accepted. Here it will be assumed that data are available at any instant from only one point in space. In the case of VLF electromagnetic wave fields observed from a single spacecraft. the assumption is reasonable because the wavelengths in the medium are generally much larger than the dimensions of any practicable antennae. which means that the wave fields are uniform on the scale of the measuring device. (This is one respect in which the case of electrostatic waves is different). While accepting this limitation for present purposes. we should remember that it could be overcome by collecting data with a suitably-disposed pair or cluster of spacecraft.

The electromagnetic field data that are available at a single point in space comprise the three orthogonal components of the wave electric field, and the three orthogonal components of the wave magnetic field, in as wide a band of frequencies as one may choose to observe. They can be measured by suitable antennae : dipoles for the electric components, loops for the magnetic components. These methods of measurement, though simple in principle, are often less so in practice, particularly for the electric field, but they have been discussed elsewhere (MOZER, 1973) and will not be considered here. The measurements will be assumed to be perfect, free both from systematic errors and from noise of technological origin, and attention will be confined to the question of how to analyze them.

Up to now, most of the methods for analyzing measurements of the six components of an electromagnetic wave field in space have been based on the assumption that the field observed in any narrow band of frequencies is locally and instantaneously like that of a single plane wave (GRARD, 1968 ; SHAWHAN, 1970 ; MEANS, 1972). In this case, the data can be interpreted readily to yield all the characteristic parameters of the wave : direction of propagation, amplitude, polarization ellipses for the electric and magnetic vectors, wave impedance. When data on all six components are available, the method of interpretation does not involve any assumption about the nature of the medium ; indeed, some of the properties of the medium can be deduced from those of the wave (STOREY, 1959). Conversely, if the properties of the medium are known, then certain items of the information obtained about the wave are redundant. Specifically, when the frequency of observation has been chosen and the direction of propagation measured, then the shapes of the polarization ellipse and the value of the wave impedance can be calculated from straightforward propagation theory. By comparing these theoretical predictions with the corresponding experimental results, the original assumption of a plane wave can be put to the test. When studying V L F waves in the magnetosphere, the polarization is particularly useful for this purpose because, under very diverse conditions, its theoretical form is almost independent of the plasma parameters. When this test is applied to discrete phenomena such as whistlers, the agreement with the theory is often satisfactory (MEANS, 1972). On the other hand, when it is applied to continuous, structureless phenomena such as VLF and ELF hiss, the results show frequent discrepancies : the observed shapes of the polarization ellipses depart from their theoretical ones, and indeed fluctuate considerably (THORNE et al. , 1973). It follows that, for such apparently random fields, the plane-wave model is much too simple, and that some kind of statistical model is required instead.

STOREY (1971), following a suggestion made privately by D.A. Gurnett, advocated that a natural noise field of this type be considered as a continuum of superposed plane waves, of different frequencies and propagated in different directions. without any mutual phase coherence. This last condition is what is meant by the statement at the beginning of the present paper, to the effect that the field in question is <u>weakly turbulent</u> : if the observations are being made within the source region. the condition requires that the field be so weak that no nonlinear effects occur ; alternatively, even when such effects occur at the source, the condition can still be satisfied if the resulting phase coherence between different frequences is destroyed by dispersion along the path to the point of observation, the latter being outside the source region. The properties of such a weakly turbulent. incoherent, or random wave field can be described statistically by a function that specifies how the wave energy density is distributed with respect to the wave-number vector $\vec{k}$, or alternatively with respect to the angular frequency ω and the direction of propagation. This concept, which is familiar to theoreticians of plasma turbulence (BEKEFI, 1966), has yet to win general acceptance among experimental space physicists.

In the light of it, the question of how to analyze measurements of the six components of a random electromagnetic wave field in a magnetoplasma reduces to the more specific one of how best to use these measurements to determine the wave distribution function.

A brief discussion of this question has been published previously (STOREY and LEFEUVRE, 1974). The present paper is intended to be the first in a series of papers that reply to it in detail. Here we are concerned with the direct problem in the statistics of random electromagnetic wave fields : namely, given a field characterized by a known distribution function, what are the statistical properties of its six components ?

The plan of the paper is as follows : section 2 presents the formal definition of the distribution function that is used to describe a random wave field statistically ; section 3 describes the standard methods that are used for the statistical description of the set of signals received on six antennae that are immersed in the field and measure its six components ; section 4 gives the solution to the direct problem (as defined above) of the relationship between the statistics of the wave field and of the received signals : finally section 5 offers some provisional conclusions.

Many assumptions and approximations are made in order to simplify the problem : all the statistical properties of the wave field are stationary in time and space ; the plasma is infinite, uniform, cold and collisionless ; the point of observation is fixed with respect to the plasma ; wave propagation is described by the magneto-ionic theory in its simplified version in which the forced motion of the ions is ignored. Their validity, and that of the basic concepts of the paper, are discussed in appendix A.

The rationalized MKSA system of units is used throughout.

2. STATISTICAL DESCRIPTION OF THE WAVE FIELD

In the kinetic theory of gases, the distribution function $F_s(t, \vec{r}, \vec{v})$ for the particle species s is defined in such a way that

$$F_s(t, \vec{r}, \vec{v})\ d^3r\ d^3v$$

is the ensemble average number of these particles, at the time t, in the volume element d^3r at the point $\vec{r}$ in coordinate space, whose velocity vectors lie in the element d^3v at the point $\vec{v}$ in velocity space.

By analogy, the distribution function $F_m(t, \vec{r}, \vec{k})$ for the wave mode m may be defined on such a way that

$$F_m(t, \vec{r}, \vec{k})\ d^3r\ d^3k$$

is the ensemble average amount of wave energy in this mode at the time t and in the volume element d^3r, due to homogeneous plane waves whose wave-number vectors lie in the element d^3k at the point $\vec{k}$ in wave-number space. However, since $\vec{k}$ and $\vec{r}$ are not independent variables, the definition of F_m involves certain conceptual difficulties that do not arise for F_s ; these are discussed in appendix A.

This wave distribution function is essentially the same as that introduced by BEKEFI (1966), in terms, however, of "light particles", which he does not define clearly. If they are what are usually called "plasmons", and each carries a quantum of energy $\hbar\omega$ where $\hbar$ is Planck's constant and ω the wave angular frequency corresponding to the vector $\vec{k}$, then Bekefi's distribution function, which is even more closely analogous to that for a species of particles of finite mass, is equal to $F_m / \hbar\omega$. Since our treatment is purely classical, we prefer to work directly in terms of the wave energy.

Note that the definition of the function F_m, in terms of the energy in a particular wave mode, involves the implicit assumption that the medium is of the type that supports wave propagation in distinct modes, i.e. that it is anisotropic. Surprisingly, this assumption very much simplifies the problem in hand. The nature of the simplification may be appreciated by considering how one would specify the intensity and state of polarization of a quasi-monochromatic wave field that is random in the restricted sense that it is spread in frequency, but not in respect of its direction of propagation which is fixed and known. In the case of an anisotropic medium, it suffices to specify the energy density in each of the possible wave modes ; thus for a cold magneto-plasma, which supports two electromagnetic wave modes, only two parameters are required. On the other hand, in the case of an isotropic medium such as free space or a non-magnetized plasma, no less than four parameters are required, for instance the Stokes parameters (BORN and WOLF, 1964). The method of analysis developed in the present paper has not yet been extended to the case of waves in isotropic media.

From here on, several assumptions will be made in order to simplify the discussion : first, the medium will be assumed to be uniform ; the wave field will be assumed to be stationary in time, and the observations to be made at a fixed point in the medium, so the variables t and $\vec{r}$ can be omitted ; also, it will be assumed provisionally that only one wave mode is present, and the subscript m will be dropped accordingly. These assumptions are discussed in the appendix A. They enable is to write the wave distribution function simply as $F(\vec{k})$.

Now when the medium is optically uniaxial, as is the case for a stationary magnetoplasma, it is more convenient to express the wave distribution function in terms of the angular frequency ω and of the direction of propagation (i.e. the direction of the $\vec{k}$ vector), as specified by the angle θ between this vector and the optic axis, and by an azimuthal angle ϕ (fig. I) ; the angle θ varies from 0 to π, and ϕ from 0 to 2π. In this representation, the distribution function will be written $F(\omega, \theta, \phi)$ and the element of wave energy is $F(\omega, \theta, \phi)$ $d^3r \; d\omega \, d\theta \, d\phi$. The relation between this and the previous form of the distribution function is

(1.1.) $$F(\omega, \theta, \phi) = V_g^{-1} \; k^2 \sin\theta \quad F(\vec{k})$$

where $k = |\vec{k}|$ and $V_g = |(\partial \omega / \partial k) e|$ is the modulus of the group velocity (BEKEFI, 1966) ; the vector $\vec{V}_g$

has the direction of the wave normal (or the opposite direction, if $(\partial\omega/\partial k)_\theta$ is negative), as distinct from the group-ray velocity which has the direction of the geometric optics ray (HELLIWELL, 1965).

The representation of the distribution function for an electromagnetic wave mode in a magnetoplasma in terms of the variables ω, θ, and ϕ is analogous to that for a charged particle species, in the presence of a magnetic field, in terms of the energy E and the pitch-angle α. There is the difference that, for low-energy particles, the distribution function has cylindrical symmetry around the magnetic field, so no azimuthal angle is required, whereas for waves, although most known source processes give rise to distributions having such symmetry. this is lost progressively by refraction as the waves are propagated away from their source region. Incidentally, it should he remembered that the distribution functions of the energetic charged particles in the earth's radiation belts are not cylindrically symmetrical at energies such that the radii of gyration are comparable with the linear scale of the gradients of the magnetic field (i.e. with an earth radius) or with the atmospheric scale height in the case of particles trapped on low L - shells (HESS. 1968).

There is one respect in which the representation $F(\omega, \theta, \phi)$ of the wave distribution function is actually better than the representation $F(\vec{k})$: namely, it is unambiguous. For electromagnetic waves in a given mode. propagated in a given direction, it is possible for a given value of k to correspond to two (or more) different values of ω. This point is illustrated in fig. 2, which shows the real branches of the dispersion curves for waves propagated perpendicularly to the magnetic field ($\theta = \pi/2$) ; the electron gyrofrequency has been taken equal to the plasma frequency. The abscissa is ω / Π, where Π is the angular plasma frequency, and the ordinate is $(c/\Pi) k$, where c is the speed of light in free space ; the dashed line is the free-space dispersion curve $\omega = ck$. Note that the function ω (k) is ambiguous for the extraordinary (X) mode, though not for the ordinary (O), whereas the function k (ω) is single-valued for both modes. In general, for a given mode, a given value of ω always corresponds to a unique value of k, and for this reason the representation F (ω, θ, ϕ) is to be preferred.

Another reason is that the experimenter, by means of narrow-band filters, can select for observation waves with frequencies ω that lie between limits of his choosing. Of course, the observed frequencies are equal to the true wave frequencies only if the point of observation is stationary with respect to the medium, because any motion gives rise to Doppler shifts proportional to k ; here this has been assumed to be the case (see appendix A.). From this point onwards, therefore, the distribution function for wave energy will be used exclusively in the representation F (ω, θ , ϕ).

In conclusion of this section, and in anticipation of those that follow, it should be stated that the term "energy" is employed in this paper in two different senses. In the context of wave propagation theory, it is used in its strict and customary sense as above. However, when discussing the analysis of data on the six field components, we shall use it also in the sense of signal theory, where for instance if x(t) and y (t) are two real time series, then $\int_0^T x^2(t)\,dt$ and $\int_0^T y^2(t)\,dt$ are their respective energies in the time interval $0 < t < T$, while $\int_0^T x(t)\,y(t)\,dt$ is their interaction energy. We shall distinguish between these two senses by using the terms "wave energy" and "signal energy" respectively ; equally we shall speak of "wave power" and "signal power".

3. STATISTICAL DESCRIPTION OF THE RECEIVED SIGNALS

3.1. Wide-band signals

When we seek to predict the statistics of the six components of a random wave field, the first question to ask is how these statistics should be described. As already mentioned, the field is pictured as being made up of a continuum of elementary plane waves of different frequencies and with random phases, propagated through the medium in different directions. We suppose that, at a fixed point in the medium, continuous measurements are made of three orthogonal electric components and three orthogonal magnetic components of the total field, using for instance three electric dipole antennae and three magnetic loop antennae. For the time being, we suppose that these measurements are made over a wide frequency band. We take a right-handed set of rectangular coordinate axes Oxyz, with Oz parallel to the steady magnetic field, and suppose for convenience that the antennae are aligned parallel to these axes. Then the experimental data consist of six wide-band signals, each proportional to one of the six axial field components.

First let us consider the individual properties of any one of these signals, separate from the other five. It is the sum of all the waveforms induced on the corresponding antenna by all the different elementary plane waves. The elementary wareform induced by any one of these waves is a sinusoid, the phase of which bears no special relationship to that of the sinusoid induced by any other such wave. Thus the total signal is the sum of an infinite number of elementary sinusoids, with infinitesimal amplitudes and random phases. It is well known that a signal of this type is a stationary Gaussian random process of zero mean value, and that, as such, its statistical properties are described completely by its mean auto-covariance function or by its mean signal power spectrum ("auto-spectrum").

As usual when discussing random processes, it is convenient to introduce the notion of a statistical ensemble.

We imagine an ensemble of random wave fields. each giving rise to a specimen of the random signal considered. We assume that, on going from one member to another of the ensemble of fields, the elementary plane waves change in amplitude, frequency, direction of propagation, and phase, while remaining consistent collectively with the given wave distribution function. On this assumption, the received signal is ergodic, so its auto-corariance function and auto-spectrum can be defined alternatively in terms of time averages or of ensemble averages.

Now let us consider the six received signals as a whole. Besides their individual properties, we need to know what relationships exist between them. These are described by the cross-covariance functions for all the fifteen possible pairs of signals, or by the corresponding cross-spectra.

Here we prefer to work with signal auto-spectra and cross-spectra, rather than to use covariance functions, because the spectra are related more simply to the wave distribution function, as we shall show later ; of course, we can get from a covariance function to the corresponding power spectrum, or vice versa, by a direct or inverse Fourier transform respectively (JENKINS and WATTS, 1969).

First, let us define a 6 x 6 <u>spectral matrix</u>, which groups all the statistical data together in a convenient way. Let E_x, E_y, E_z be the axial components of the electric field of the wave at the point of observation, and H_x, H_y, H_z the components of the wave magnetic field. From these variables, we define a generalized electric field vector $\vec{\mathcal{E}}$ as follows :

(1.2a) $\mathcal{E}_1 = E_x \qquad \mathcal{E}_2 = E_y \qquad \mathcal{E}_3 = E_z$

(1.2b) $\mathcal{E}_4 = Z_0 H_x \qquad \mathcal{E}_5 = Z_0 H_y \qquad \mathcal{E}_6 = Z_0 H_z$

where Z_0 is the wave impedance of free space. (Note : another way of defining a generalized electric field vector is mentioned in appendix B). This step in the argument is just a convenient way of regrouping the data, and has no physical significance. Let $\mathcal{E}_i$ be any component of this vector, the subscript i running from 1 to 6. We now introduce the 6 x 6 spectral matrix, of which any element $S_{ij}(\omega)$ is either the auto-power spectrum of the field component $\mathcal{E}_i$ (if i = j), or the cross-power spectrum of the components $\mathcal{E}_i$ and $\mathcal{E}_j$ (if $i \neq j$).

These are "two-sided" signal power spectra : that is, they have values at negative as well as at positive frequencies. They are the Fourier transforms of the corresponding covariance functions. Since the latter functions are purely real, it follows that $S_{ij}(\omega) = S^*_{ij}(-\omega)$, where the asterisk denotes the complex conjugate ; therefore it suffices to quote the expressions for the spectra at positive frequencies only.

At any one frequency ω, the spectral matrix contains 36 items of statistical information about the field. It might be thought that the number of independent items is less than this, because the matrix is Hermitian : $S_{ij}(\omega) = S^*_{ji}(\omega$ Hence here are only 6 independent auto-spectra and 15 cross-spectra. Note, however, that while the auto-spectra are purely real, since the auto-covariance functions are symmetrical, the cross-spectra are complex, since the cross-covariance functions are asymmetrical in general. Moreover the real and imaginary parts of the cross-spectra are mutually independent, i.e. there is no general relation between them. Hence the number of independent items of information is 36 as stated.

3.2. Narrow-band signals

Now let us consider the case where the 6 data signals are received in a band of width $\Delta\omega$, centred on some frequency ω_o. We suppose moreover that this band is narrow, by which we mean two things : firstly, that $\Delta\omega \ll \omega_o$; secondly, that all the 36 signal power spectra $S_{ij}(\omega)$ are essentially constant and equal to $S_{ij}(\omega_o)$ throughout this band.

Then clearly the information contained in the received signals relates only to the values of the 36 quantities $S_{ij}(\omega_o)$; the question is how to extract the relevant information from the data.

Let $X_i(t)$ be the real narrow-band signal obtained by filtering the original wide-band signal $E_i(t)$ through the receiver pass-band. Let $\chi_i(t)$ be the corresponding analytic signal. i.e. the representation of $X_i(t)$ by a complex exponential (HELLSTROM, 1968). We now define the <u>covariance matrix</u> $\{c_{ij}\}$ of the set of 6 narrow-band real signals, with (1.3) $$c_{ij} = \langle \chi_i(t) \; \chi_j^*(t) \rangle$$

where the triangular brackets denote the (BORN and WOLF, 1964) ensemble average or "expected value" of the product. If the signals are stationary and ergodic, as has been assumed, then these same quantities are given by the corresponding time averages in the limit of very long times. A practical method for estimating the elements c_{ij} of the covariance matrix in this way has been described by MEANS (1972).

In order to relate these elements to those of the spectral matrix at the frequency ω_o. it is necessary to define the bandwidth $\Delta\omega$ more precisely. If the transfer function of the receiver is $Y(\omega)$, between the point where the signal $E_i(t)$ enters it and that at which $X_i(t)$ emerges, and if, at positive frequencies, this function takes the form of a single peak with its summit at ω_o, then we define

(1.4) $$\Delta\omega = \frac{\int_0^\infty |Y(\omega)|^2 \, d\omega}{|Y(\omega_o)|^2}$$

In engineering parlance, this quantity is known as the <u>noise bandwidth</u> (BENDAT and PIERSOL, 1971).

With $\Delta\omega$ thus defined, it follows from elementary considerations of signal energy conservation that

(1.5) $$c_{ij} = 4 \; \Delta\omega \; |Y(\omega_o)|^2 \; S_{ij}(\omega_o)$$

The factor 4 stems from the fact that, at positive frequencies, the amplitudes of the Fourier components of $\chi_i(t)$ are twice those of the corresponding components of $X_i(t)$.

4. DIRECT RELATIONSHIP BETWEEN WAVE AND SIGNAL STATISTICS

The problem is to relate the spectral matrix, which describes the statistics of the 6 received signals, to the distribution function, which describes those of the random wave field. In order to do this, we shall show how each of these entities is made up of contributions from all the various elementary waves ; we begin with the case where these waves all belong to the same magneto-ionic mode.

First let us consider one such wave, which we shall distinguish from its fellows by placing the subscript 1 in front of every symbol that refers to it. The i'th component of its generalized electric field, which will be called ${}_1\varepsilon_i\ (t, \vec{r})$, varies sinusoidally as a function of time and of spatial position :

$$(1.6)\qquad {}_1\varepsilon_i\ (t, \vec{r}) \quad = \quad \mathrm{Re}\ \{\ {}_1e_i\ \exp\left[\ i\ ({}_1\omega t\ -\ {}_1\vec{k}.\vec{r})\right]\}$$

On the right-hand side of this equation, the symbol Re denotes the extraction of the real part, ${}_1e_i$ is the complex amplitude, the angular frequency ${}_1\omega$ is real and positive, and the wave-number vector ${}_1\vec{k}$ is real because the medium is assumed to be lossless, and also because only homogeneous waves are being considered, as befits a situation in which there are no boundaries.

Now if, in coordinate space, the wave has an electromagnetic energy density ${}_1\rho$, then its distribution function, which we shall call ${}_1f\ (\vec{k})$, is a 3-dimensional Dirac distribution of strength ρ at the point ${}_1\vec{k}$ in $\vec{k}$ space :

$$(1.7)\qquad {}_1f\ (\vec{k}) \quad = \quad {}_1\rho \quad \delta(\vec{k}\ -\ {}_1\vec{k})$$

The alternative form of this distribution function is

$$(1.8)\qquad {}_1f\ (\ \omega,\ \theta.\phi\ ) \quad = \quad {}_1\rho \quad \delta(\omega\ -\ {}_1\omega\ )\ \delta(\theta\ -\ {}_1\theta\ )\ \delta(\phi -{}_1\phi$$

where ${}_1\theta$ and ${}_1\phi$ are the angles that specify the direction of ${}_1\vec{k}$. The equivalence of these two expressions is evident, because in both cases the wave energy per unit volume of coordinate space is given by a triple integral over the distribution function :

(1.9) $\quad {}_1\rho = \iiint {}_1f\,(\vec{k})\,d^3k = \iiint {}_1f\,(\omega,\theta,\phi)\,d\omega\,d\theta\,d\phi$

In spite of superficial appearances, the expressions (1.7) and (1.8) are consistent with (1.1) ; the algebraic factors on the right-hand side of (1.1) are incorporated in the delta-functions on the right-hand side of (1.8).

The elements of the spectral matrix for this same elementary wave are, for ω positive,

(1.10) $\quad {}_1s_{ij}\,(\omega) = \frac{\pi}{2}\;{}_1e_i\;{}_1e_j^{*}\;\delta(\omega - {}_1\omega) \qquad (\omega > o)$

Probably the simplest way to derive this result is to obtain the expression for the cross-covariance function of the periodic signals ${}_1\varepsilon_i\,(t)$ and ${}_1\varepsilon_j\,(t)$ at a fixed point $\vec{r}$, and then to take its Fourier transform.

Now. instead of considering just one elementary wave, let us consider the subset of elementary waves for which the parameters ${}_1\omega$, ${}_1\theta$. and ${}_1\phi$ lie respectively in the ranges from ω to $\omega + d\omega$, from θ to $\theta + d\theta$, and from ϕ to $\phi + d\phi$. Their contribution to the wave energy per unit volume of coordinate space is $\sum_1 {}_1\rho$, where the sum is taken over this subset. Bv definition, the distribution function for the random field is related to this quantity as follows :

(1.11) $\quad F\,(\omega,\theta,\phi)\,d\omega\,d\theta\,d\phi = \left\langle \sum_1 {}_1\rho \right\rangle$

Here the brackets on the right-hand side denote the ensemble average.

Similarly, the elementary waves in this subset make the following contribution to the signal energy of interaction between the field components E_i and E_j, in the frequency range from ω to $\omega + d\omega$:

(1.12) $\quad \sigma_{ij}\,(\omega)\,d\omega = \frac{\pi}{2}\left\langle \sum_1 {}_1e_i\;{}_1e^{*}_j \right\rangle \qquad (\omega > o)$

At this point we introduce a crucial idea : for given values of ω, θ, and ϕ. the wave energy density ρ is proportional to the square of the amplitude of any one field component ; the same is true of the product ${}_1e_i\;{}_1e^{*}_j$; therefore the ratio

$$(1.13) \qquad a_{ij} = \frac{e_i \quad e_j^{*}}{\rho}$$

is independent of the amplitude, and is essentially the same for all the elementary waves in the subset defined above, i.e. it is independent of 1. With all the possible pairs of values of the indices i and j, the identity (1.13) defines 36 such ratios. The expressions for them, as functions of ω, θ, and ϕ and of the characteristic parameters of the plasma, are given by the propagation theory for the type of wave considered, i.e. by the magneto-ionic theory in the present instance ; they are quoted in the appendix B. It is at this point that our knowledge of the properties of the medium enters into the argument.

This idea enables us to write

$$(1.14) \qquad \sigma_{ij} \; (\omega) \quad d\omega = \frac{\pi}{2} \quad a_{ij} \; (\omega, \theta, \phi) \left\langle \sum_{1} \; {}_{1}\rho \right\rangle \quad (\omega > o)$$

$$(1.15) \qquad = \frac{\pi}{2} \quad a_{ij} \; (\omega, \theta, \phi) \quad F \; (\omega, \theta, \phi) \; d\omega \; d\theta \; d\phi \; (\omega > o)$$

with (1.15) following from (1.11), and enabling us to cancel $d\omega$ on both sides.

Finally, in order to obtain the total auto-spectral ($i = j$) or cross-spectral ($i \neq j$) signal energy in the frequency range from ω to $\omega + d\omega$, it suffices to sum over the complete set of elementary waves in this range, with all possible directions of propagation. In other words, (1.15) must be integrated over the full ranges of the angles θ and ϕ. Then, dividing by $d\omega$, we obtain

$$(1.16) \qquad S_{ij} \; (\omega) = \frac{\pi}{2} \int_0^{\pi} \int_0^{2\pi} a_{ij} \; (\omega, \theta, \phi) \quad F \; (\omega, \theta, \phi) \; d\theta \; d\phi$$

for ω positive. This is the required result, for the special case where only one of the two magneto-ionic modes are present.

In the general case where both modes are present simultaneously, each spectral matrix element is just the sum of the contributions from the two modes :

$$(1.17) \qquad S_{ij} \; (\omega) = \frac{\pi}{2} \sum_{m} \int_0^{\pi} \int_0^{2\pi} a_{ijm} \; (\omega, \theta, \phi) \; F_m \; (\omega, \theta, \phi) \; d\theta \; d\phi$$

where the subscript m denotes the wave mode.

From their definitions, it follows that the spectral matrix S_{ij}, together with the kernels a_{ij} for the two modes, are tensors of rank 2.

5. CONLUSION

Subject to the assumptions made in this paper. equation (1.17) is the solution for the direct problem of determining the statistics of the received signals when those of the wave field are known.

As such, it already provides a weak basis for comparison between theory and experiment in the study of the origin of certain natural random electromagnetic wave fields, such as those of magnetospheric VLF and ELF hiss. If the theory predicts the wave distribution function explicitly, and if an accurate estimate of the spectral matrix is available from experimental data, then the two can be compared by means of this equation.

More often, however, the task of comparing theory and experiment is less simple. The data may be degraded by noise, or the time of observation too short to yield good statistics. The theory may involve one or more unknown parameters, which have to be adjusted in order to make its predictions agree with the data as closely as possible. There may be several competing theories, none of which explains the data perfectly, and then one wants to know whether any of these theories is acceptable, and if so, which is the most plausible. Or again, one may be interested in analyzing some experimental data to find out whatever one can about the wave distribution function, in the absence of any theoretical model. All there are different forms of the inverse problem, which is more difficult and will be treated in subsequent papers.

At the same time, further work is needed on the direct problem, in order to free the theory from the limitations accepted in this paper. For instance, before it can be applied to space-probe data on the random field of Alfvén waves in the solar wind, the theory must be generalized to take account of the motion of the point of observation with respect to the medium, and to make use of data on the wave-induced fluctuations of plasma velocity. The application to random wave fields other than electromagnetic must be considered. The need to study the simple special case of isotropic media has been noted already, in section 2. Another interesting special case is that in which the wave field is random in space but not in time, an example being the field created when an initially plane monochromatic wave traverses a spatially irregular but temporally stationary medium. The study of this case would begin to forge the links between two domains hitherto independent, namely previous work on the propagation of deterministic wave fields through random media, and the present work on the analysis of measurements of random wave fields in uniform media.

A C K N O W L E D G E M E N T S

The authors are most grateful to Professor D.A. Gurnett for suggesting this problem, in a private communication to L.R.O. Storey in 1965. The work was begun in 1972 when one of us (L.R.O.S) was a guest worker at the Institute for Extraterrestrial Physics of the Max-Planck Institute for Physics and Astrophysics, Garching (German Federal Republic), at the kind invitation of Dr. G. Haerendel. Our thanks are also due to our colleague Dr. M.R. Feix for helpful criticism.

REFERENCES

BEKEFI, G. "Radiation Processes in Plasmas", Wiley, New York, 1966.

BENDAT, J.S. and A.G. PIERSOL, "Random Data : Analysis and Measurement Procedures". Wiley, New York, 1971.

BORN, M. and E. WOLF, "Principles of Optics" (4th. ed.), Pergamon, London, 1970.

BUDDEN, K.G. "Radio Waves in the Ionosphere", University Press, Cambridge (G.B.) 1961.

GRARD, R. "Interprétation de mesures de champ electromagnétique T.B.F. dans la magnétosphère", Ann. Géophys. 24, 955-971, 1968.

HELLIWELL. R.A. "Whistlers and related Ionospheric Phenomena". Stanford University Press. Stanford. 1965.

HELLSTROM. C.W. "Statistical Theory of Signal Detection" (2 nd.ed.). Pergamon London, 1968.

HESS. W.N. "The Radiation Belt and the Magnetosphere". Blaisdell. Toronto, 1968.

JENKINS. G.M. and D.G. WATTS. "Spectral Analysis and its Applications". Holden-Day, San Francisco. 1969.

LYONS. L.R., THORNE. R.M. and C.F. KENNEL. "Pitch-angle diffusion of radiation-belt electrons within the plasmasphere", J. Geophys. Res. 77; 3455-3474. 1972.

MEANS. J.D. "The use of the three-dimensional covariance matrix in analyzing the polarization properties of plane waves". J. Geophys. Res. 77. 5551 - 5559, 1972.

MOZER. F.S. "Analysis of techniques for measuring DC and AC electric fields in the magnetosphere". Space Sci. Rev. 14, 272-313. 1973.

RATCLIFFE. J.A. "The Magneto-ionic Theory and its Applications to the Ionosphere". University Press, Cambridge (G.B.). 1959.

RUSSELL. C.T. Mc. PHERRON, R.L., and P.J. COLEMAN, "Fluctuating magnetic fields in the magnetosphere : 1- ELF and VLF fluctuations". Space Sci. Rev. 12. 810-856, 1972.

SCHULZ. M. and L.J. LANZEROTTI. "Particle Diffusion in the Radiation Belts". Springer-Verlag, Berlin, 1974.

SHAWHAN. S.D. "The use of multiple receivers to measure the wave characteristics of very-low-frequency noise in space". Space Sci. Rev. 10, 689-736. 1970.

STOREY, L.R.O. "A method for measuring local electron density from an artificial satellite". J. Res. Nat. Bur. Stand. 63 D. 325-340, 1959.

STOREY, L.R.O. "Electric field experiments : - alternating fields", in "The ESRO geostationary magnetospheric satellite". European Space Research Organisation, Neuilly-sur-Seine, Report n° SP-60, 267-279, 1971.

STOREY. L.R.O. and F. LEFEUVRE. "Theory for the interpretation of measurements of a random electromagnetic wave field in space". in "Space Research XIV" (Eds : M.J. RYCROFT and R.D. REASENBERG) Akademie-Verlag, Berlin, 381-386, 1974.

THORNE. R.M.. SMITH. E.J. , BURTON. R.K. and R.E. HOLZER. "Plasmaspheric hiss". J. Geophys. Res. 78. 1581-1596, 1973.

APPENDIX A : Validity of the basic concepts and assumptions.

In section 2, it is assumed that the random wave field is statistically stationary in space and in time, and that only one wave mode is present. These assumptions are less restrictive than might appear at first sight. The conditions under which they may be considered to be satisfied adequately are discussed below.

Approximate spatial uniformity is required only to ensure that the structure of a single wave be very little different from what it would be in an infinite uniform medium, with properties like those of the real medium at the point considered. For this purpose, it is necessary that the medium be uniform on a scale much larger than a wavelength. The residual large-scale non-uniformity limits the meaningful detail that the distribution function can exhibit : if L is the distance over which the medium is substantially uniform, then this function can be specified with a resolution, in any component of $\vec{k}$, of the order of $\Delta k = 2\pi/L$.

Approximate temporal stationarity of the field is needed to provide reasonable data statistics, which means that the received signals must be statistically stationary over time intervals much longer than a wave period. If they are in fact so for a time T, then the distribution function can be measured with a resolution in frequency of the order of $\Delta\omega = 2\pi/T$ (moreover, the point of observation is effectively stationary if all Doppler shifts due to its motion are less than $\Delta\omega$).

Under the conditions enunciated in the previous two paragraphs, it is possible to define a wave distribution function that varies with time t and with spatial position $\vec{r}$, as was mentioned in section 2. For this purpose, the fields must be truncated by means of a suitable "window" function, of linear dimensions L and duration T, centred on the point-instant $(\vec{r}, t)$ considered, before taking their Fourier transforms ; the choice of L and T also limits the meaningful resolution of F in wave number and in frequency respectively.

These same conditions justify the use of geometric optics to determine how a given distribution function evolves in time and in space (BEKEFI, 1966). In the present paper, such questions have been avoided by the assumption of stationarity.

Finally, the assumption that only one wave mode is present is satisfied over wide frequency ranges, in particular in the range from the lower hybrid frequency up to the electron gyro-frequency, in which only the whistler (ordinary) mode is propagated. Even at frequencies where both modes can be propagated, natural source mechanisms are likely to excite one much more strongly than the other. Finally, as we saw in section 4, this assumption can be dropped if necessary without greatly complicating the theory.

APPENDIX B : expressions for the kernels a_{ij} and b_{ij}

The 36 functions a_{ij} (ω,θ,ϕ), which are the kernels of the set of integral equations (1.16), were defined by (1.13) in the course of an analysis based on a right-handed rectangular coordinate system Oxyz, with Oz parallel to the steady magnetic field. For pratical purposes, algebraic expressions are required for these weighting functions in terms of the variables ω, θ and ϕ, and of the characteristic parameters of the medium.

However, since the medium has been assumed to be uniaxial, the required expressions would have been simpler if the analysis had been developed using a "circularly-polarized" coordinate system (STIX, 1962), in which the complex amplitudes of the sinusoidal components of the electric field vector $\vec{e}$ of an elementary plane wave are

(1.18) $e_R = e_x + ie_y \qquad e_L = e_x - ie_y \qquad e_P = e_z$

and similarly for the magnetic vector $\vec{h}$. The subscripts R, L, and P stand for "right-handed", "left-handed", and "parallel" respectively. We now define a generalized electric field vector $\vec{f}$, the 6 components of which have the complex amplitudes

(1.19a) $f_1 = e_R \qquad f_2 = e_L \qquad f_3 = e_P$

(1.19b) $f_4 = Z_o h_R \qquad f_5 = Z_o h_L \qquad f_6 = Z_o h_P$

Then, redeveloping the analysis along the lines of section 4, we are led to define a new set of 36 kernels,

(1.20) $$b_{ij} = \frac{f_i f_j^*}{\rho}$$

for which we shall now quote the expressions.

First, however, we must define the plasma parameters. In this we shall follow STIX (1962), with some minor differences : in particular, we suppose that the plasma contains only electrons and positive ions, and that the latter, of which there may be several species present, are all singly charged. Denoting the different types of charged particle - including the electrons - by a subscript k, let n_k be the number density for each type, m_k the mass per particle, and q_k the charge per particle. For each type of particle, we now define two characteristic frequencies : the angular plasma frequency Π_k, such that

(1.21) $$\Pi_K^2 = \frac{n_k q_k^2}{\varepsilon_0 m_k}$$

where ε_0 is the electric permittivity of free space, and the angular gyrofrequency

(1.22) $$\Omega_k = - \frac{q_k B_0}{m_k}$$

where B_0 is the magnitude of the induction vector of the steady magnetic field. The minus sign has been put into this definition so as to give Ω_k the sign of the natural sense of gyration of the particles around the field (e.g. positive for the electrons, which gyrate in a right-handed sense). Lastly, we define three dimensionless quantities :

(1.23a) $$R = 1 - \sum_k \frac{\Pi_k^2}{\omega(- \Omega_k)}$$

(1.23b) $$L = 1 - \sum_k \frac{\Pi_k^2}{\omega(\omega + \Omega_k)}$$

(123c) $$P = 1 - \sum_k \frac{\Pi_k^2}{\omega 2}$$

These quantities are identical with those that Stix denotes by the same symbols ; in the expressions that follow , they represent the properties of the plasma.

From these we must define several derived quantities, the first of which is the phase refractive index n for electromagnetic waves in either of the two magneto-ionic modes. This quantity is the solution of the equation

(1.24) $$\tan^2\theta = - \frac{P(n^2-R)\ (n^2-L)}{(Sn^2-RL)\ (n^2-P)}$$

which is a quadratic in n^2 with purely real roots (STIX, 1962).

For a given θ and given plasma parameters. (1.24) may have two, one, or no roots with n^2 positive. Only such positive roots, which correspond to freely propagated modes, are to be considered here ; negative roots correspond to evanescent modes. Moreover. for a given positive n^2, only the positive value of n will be considered, the negative value corresponding to a wave propagated in the opposite direction. On the questions of the nomenclature for the two modes, and of their correspondence with the roots, the reader should consult the standard works on the magneto-ionic theory (RATCLIFFE 1959, BUDDEN 1961).

One more dimensionless quantity remains to be defined. It involves the group refractive index

$$(1.25) \qquad n_g = \left[\frac{\partial}{\partial\omega}(\omega n)\right]^{-1}$$

and two other parameters :

$$(1.26) \quad \lambda = 2n\left\{(P-n^2)\left[1+\frac{(P-L)(R-n^2)^2}{(P-R)(L-n^2)^2}\right] - (R-n^2)\left[1+\frac{(P-L)(R-n^2)}{(P-R)(L-n^2)}\right]\right\}\cos^2\theta$$

$$(1.27) \quad \mu = \frac{n(R-L)^2}{(P-R)}\frac{(P-n^2)^2}{(L-n^2)^2}\sin^2\theta$$

In these terms, the required quantity is

$$(1.28) \qquad \xi = 8\, n_g\left[\varepsilon_0(\lambda+\mu)\right]^{-1}$$

We are now in a position to list the 36 kernels b_{ij}. In point of fact, it suffices to list the 6 kernels with $i=j$, and 15 kernels with $i \neq j$, since the remaining 15 are given by the relation $b_{ji} = b^*_{ij}$, which follows from their definition in (1.20). Here are the 21 expressions :

$$(1.29) \quad b_{11} = \frac{(P-n^2)^2}{P-R}\,\xi$$

$$(1.30) \quad b_{12} = \frac{(R-n^2)(P-n^2)^2}{(L-n^2)\,(P-R)}\,\xi\,\exp(2i\phi)$$

$$(131) \quad b_{13} = \frac{(R-n^2)(P-n^2)}{P-R}\,\xi\cot\theta\exp(i\phi)$$

(1.32) $b_{14} = -\, i\, \left[n(P-n^2)\xi \cos\theta\right]$

(1.33) $b_{15} = i\, \left[\dfrac{n(R-n^2)(P-L)(P-n^2)}{(L-n^2)(P-R)}\, \xi \cos\theta \exp(2i\phi)\right]$

(1.34) $b_{16} = -i\left[\dfrac{n(R-L)(P-n^2)^2}{2(P-R)(L-n^2)}\, \xi \sin\theta \exp(i\phi)\right]$

(1.35) $b_{22} = \dfrac{(P-n^2)^2(R-n^2)^2}{(P-R)(L-n^2)^2}\, \xi$

(1.36) $b_{23} = \dfrac{(P-n^2)(R-n^2)^2}{(P-R)(L-n^2)}\, \xi \cot\theta \exp(-i\phi)$

(1.37) $b_{24} = -i\left[\dfrac{n(P-n^2)(R-n^2)}{L-n^2}\, \xi \cos\theta \exp(2i\phi)\right]$

(1.38) $b_{25} = i\left[\dfrac{n(P-n^2)(P-L)(R-n^2)^2}{(P-R)(L-n^2)^2}\, \xi \cos\theta\right]$

(1.39) $b_{26} = -i\left[\dfrac{n(R-L)(R-n^2)(P-n^2)^2}{2(P-R)(L-n^2)^2}\, \xi \sin\theta \exp(-i\phi)\right]$

(1.40) $b_{33} = \dfrac{(R-n^2)^2}{P-R}\, \xi \cot^2\theta$

(1.41) $b_{34} = -\, i\left[n(R-n^2)\, \xi \cos\theta \cot\theta \exp(-i\phi)\right]$

(1.42) $b_{35} = i\left[\dfrac{n(P-L)(R-n^2)^2}{(P-R)(L-n^2)}\, \xi \cos\theta \cot\theta \exp(i\phi)\right]$

(1.43) $b_{36} = -i\left[\dfrac{n(R-L)(R-n^2)(P-n^2)}{2(P-R)(L-n^2)}\, \xi \cos\theta\right]$

(1.44) $b_{44} = n^2(P-R)\, \xi \cos^2\theta$

(1.45) $b_{45} = -\left[\dfrac{n^2(P-L)(R-n^2)}{L-n^2}\, \xi \cos^2\theta \exp(2i\phi)\right]$

(1.46) $b_{46} = \dfrac{n^2(R-L)(P-n^2)}{2(L-n^2)}\, \xi \sin\theta \cos\theta \exp(i\phi)$

$$(1.47)\quad b_{55} = - \frac{n^2(P-L)^2\ (R-n^2)^2}{(P-R)(L-n^2)^2}\ \xi\ \cos^2\theta$$

$$(1.48)\quad b_{56} = - \frac{n^2(P-L)(R-L)(P-n^2)(R-n^2)}{2(P-R)(L-n^2)^2}\ \xi\sin\theta\ \cos\theta\ \exp\ (-i\phi)$$

$$(1.49)\quad b_{66} = \frac{n^2(P-n^2)^2\ (R-L)^2}{4(P-R)\ (L-n^2)^2}\ \xi\ \sin^2\theta$$

The derivation of these results. which is straightforward but lengthy, will be published elsewhere. They apply to both magneto-ionic modes ; however, the values of n and of ξ are different for the two modes. so in general the b_{ij} are different also.

These kernels can be used directly for interpreting data on random wave fields, provided that the 6 field components are transformed to the circularly-polarized coordinate system before being used to estimate their spectral matrix. On the other hand. if one prefers to work with data in the rectangular system Oxyz, then one requires the kernels a_{ij}. which are given in terms of the b_{ij} by the following expressions :

$$(1.50)\quad a_{11} = \frac{1}{4}(b_{11} + b_{12} + b21 + b_{22})$$

$$(151)\quad a_{12} = \frac{i}{4}(b_{11} - b_{12} + b21 - b_{22})$$

$$(1.52)\quad a_{13} = \frac{1}{2}(b_{13} + b_{23})$$

$$(1.53)\quad a_{22} = \frac{1}{4}(b_{11} - b_{12} - b_{21} + b_{22})$$

$$(1.54)\quad a_{23} = \frac{i}{2}(b_{23} - b13)$$

$$(1.55)\quad a_{33} = b_{33}$$

The 3 other kernels with i and j $\leqslant$3 can be found by using the relation $a_{ji} = a_{ij}^*$. From this set of 9 kernels, the remaining 27 with i and/or j $\geqslant$ 4 can be obtained by adding 3 to the first and/or second subscript of every term in each expression. These results follow quite simply from the definitions (1.18) and (1.19).

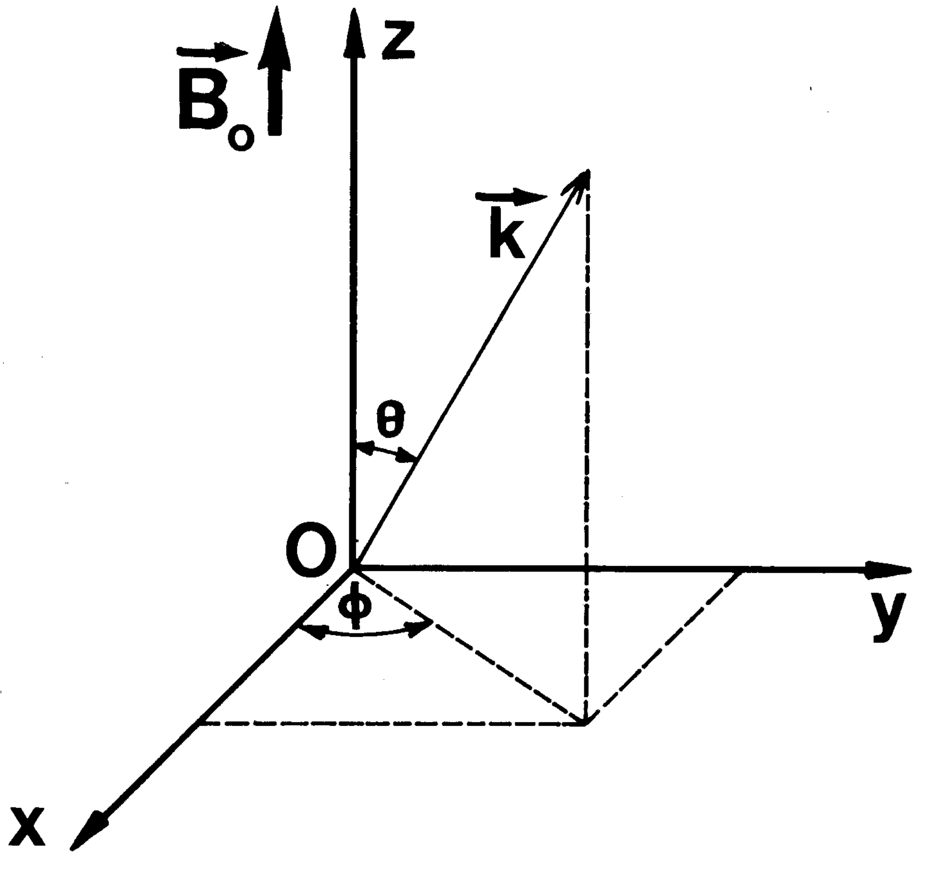

FIG. 1: COORDINATE SYSTEM USED IN THE DEFINITION OF THE WAVE DISTRIBUTION FUNCTION.

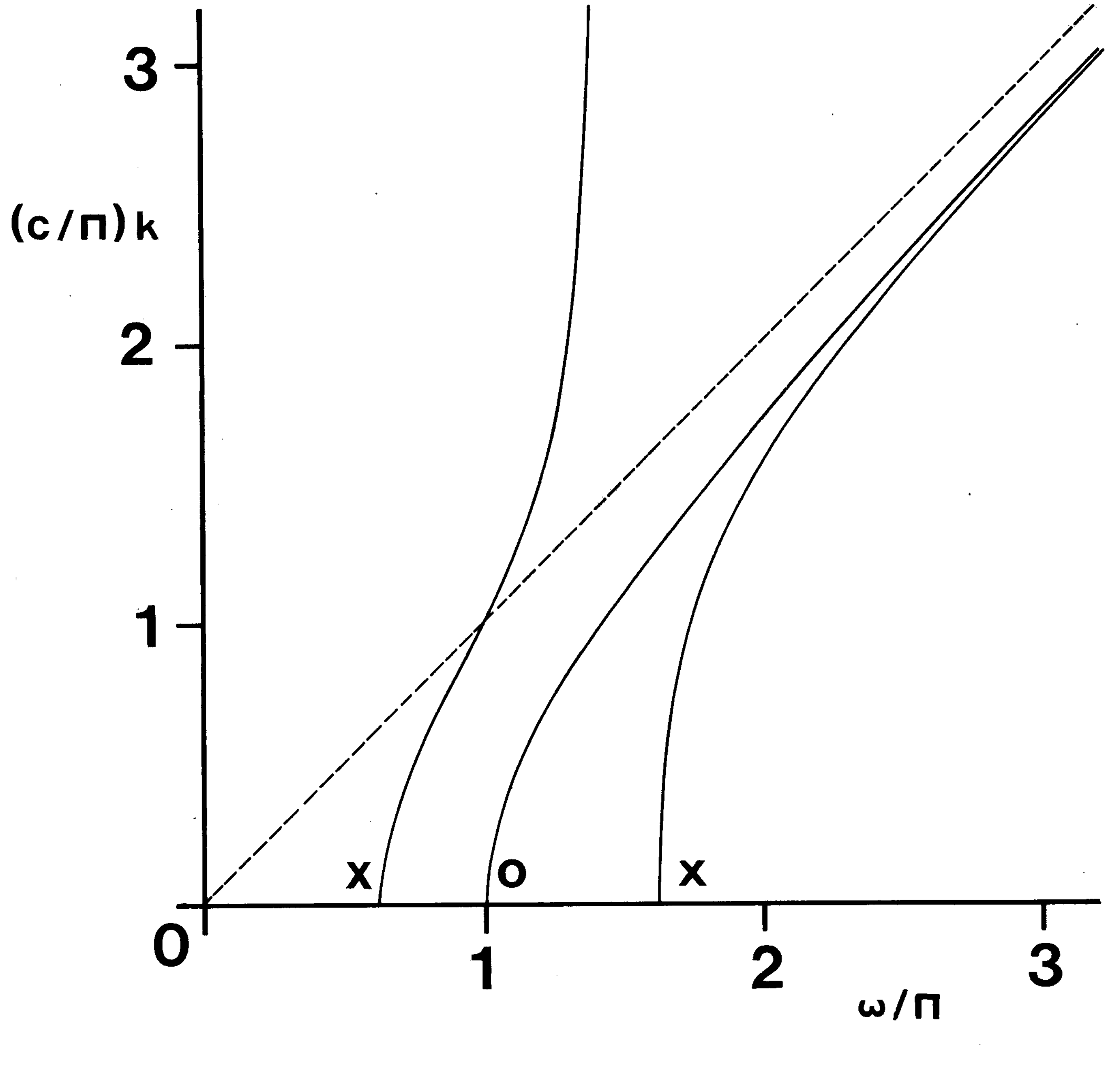

FIG. 2: DISPERSION CURVES FOR PERPENDICULAR PROPAGATION.

CRPE
Centre de Recherches en Physique de l'Environnement terrestre et planétaire

Avenue de la Recherche scientifique
45045 ORLEANS CEDEX

Département PCE
Physique et Chimie de l'Environnement

Avenue de la Recherche scientifique
45045 ORLEANS CEDEX

Département ETE
Etudes par Télédétection de l'Environnement

CNET - 38-40 rue du général Leclerc
92131 ISSY-LES-MOULINEAUX

CNET imp 75/1163