Resource allocation strategies for in-network stream processing

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Abstract

In this paper we consider the operator mapping problem for in-network stream processing applications. In-network stream processing consists in applying a tree of operators in steady-state to multiple data objects that are continually updated at various locations on a network. Examples of in-network stream processing include the processing of data in a sensor network, or of continuous queries on distributed relational databases. We study the operator mapping problem in a "constructive" scenario, i.e., a scenario in which one builds a platform dedicated to the application buy purchasing processing servers with various costs and capabilities. The objective is to minimize the cost of the platform while ensuring that the application achieves a minimum steady-state throughput.
The first contribution of this paper is the formalization of a set of relevant operator-placement problems as linear programs, and a proof that even simple versions of the problem are NP-complete. Our second contribution is the design of several polynomial time heuristics, which are evaluated via extensive simulations and compared to theoretical bounds for optimal solutions.

Keywords: in-network stream processing, trees of operators, operator mapping, optimization, complexity results, polynomial heuristics.

Résumé

Dans ce travail nous nous intéressons au problème de placement des applications de traitement de flux en réseau. Ce problème consiste à appliquer en régime permanent un arbre d'opérateurs à des données multiples qui sont mise à jour en permanence dans les différents emplacements du réseau. Le traitement de données dans les réseaux de détecteurs ou le traitement de requêtes dans les bases de données relationnelles sont des exemples d'application. Nous étudions le placement des opérateurs dans un scénario "constructif", i.e., un scénario dans lequel la plate-forme pour l'application est construite au fur et à mesure en achetant des serveurs de calcul ayant un vaste choix de coûts et de capacités. L'objectif est la minimisation du coût de la plate-forme en garantissant que l'application atteint un débit minimal en régime permanent.
La première contribution de cet article est la formalisation d'un ensemble pertinent de problèmes opérateur-placement sous forme d'un programme linéaire ainsi qu'une preuve que même les instances simples du problème sont NP-complètes. La deuxième contribution est la conception de plusieures heuristiques polynomiales qui sont évaluées a l'aide de simulations extensives et comparées aux bornes théoriques pour des solutions optimales.

Mots-clés: traitement de flux en réseau, arbres d'opérateurs, placement d'opérateurs, optimisation, résultats de complexité, heuristiques polynomiales.

1 Introduction

In this paper we consider the execution of applications structured as trees of operators. The leaves of the tree correspond to basic data objects that are spread over different servers in a distributed network. Each internal node in the tree denotes the aggregation and combination of the data from its children, which in turn generate new data that is used by the node's parent. The computation is complete when all operators have been applied up to the root node, thereby producing a final result. We consider the scenario in which the basic data objects are constantly being updated, meaning that the tree of operators must be applied continuously. The goal is to produce final results at some desired rate.

The above problem, which is called *stream processing* [1], arises in several domains. An important domain of application is the acquisition and refinement of data from a set of sensors [2, 3, 4]. For instance, [2] outlines a video surveillance application in which the sensors are cameras located in different locations over a geographical area. The goal of the application could be to show an operator monitored area in which there is significant motion between frames, particular lighting conditions, and correlations between the monitored areas. This can be achieved by applying several operators (filters, image processing algorithms) to the raw images, which are produced/updated periodically. Another example arises in the area of network monitoring [5, 6, 7]. In this case the sources of data are routers that produce streams of data pertaining to packets forwarded by the routers. One can often view stream processing as the execution of one of more "continuous queries" in the relational database sense of the term (e.g., a tree of join and select operators). A continuous query is applied continuously, i.e., at a reasonably fast rate, and returns results based on recent data generated by the data streams. Many authors have studies the execution of continuous queries on data streams [8, 9, 10, 11, 12].

In practice, the execution of the operators on the data streams must be distributed over the network. In some cases, for instance in the aforementioned video surveillance application, the cameras that produce the basic objects do not have the computational capability to apply any operator effectively. Even if the servers responsible for the basic objects have sufficient capabilities, these objects must be combined across devices, thus requiring network communication. A simple solution is to send all basic objects to a central compute server, but it proves unscalable for many applications due to network bottlenecks. Also, this central server may not be able to meet the desired target rate for producing results due to the sheer amount of computation involved. The alternative is then to distribute the execution by mapping each node in the operator tree to one or more compute servers in the network (which may be distinct or co-located with the devices that produce/store and update the basic objects). One then talks of *in-network* stream processing. Several in-network stream processing systems have been developed [13, 14, 15, 16, 17, 6, 18, 19]. These systems all face the same question: to which servers should one map which operators?

In this paper we address the operator-mapping problem for in-network stream processing. This problem was studied in [2, 20, 21]. The work in [20] studied the problem for an ad-hoc objective function that trades off application delay and network bandwidth consumption. In this paper we study a more general objective function. We first enforce the constraint that the rate at which final results are produced, or *throughput*, is above a given threshold. This corresponds to a Quality of Service (QoS) requirement of the application, which is almost always desirable in practice (e.g., up-to-date results of continuous queries must be available at a given frequency). Our objective is to meet this constraint while minimizing the "overall cost", that is the amount of resources used to achieve the throughput. For instance, the cost could be simply the total number of compute servers, in the case when all servers are identical and network bandwidth is assumed to be free.

We study several variations of the operator-mapping problem. Note that in all cases basic objects may be replicated at multiple locations, i.e., available and updated at these locations. In terms of the computing platform one can consider two main scenarios. In the first scenario, which we term "constructive", the user can build the platform from scratch using off-the-shelf components, with the goal of minimizing monetary cost while ensuring that the desired throughput is achieved. In the second scenario, which we term "non-constructive", the platform already exists and the goal is to use the smallest amount of resources in this platform while achieving the desired throughput. In this case we consider platforms that are either fully homogeneous, or with a homogeneous network but heterogeneous compute servers, or fully heterogeneous. In terms of the tree of operators, we consider general binary trees and discuss relevant special cases (e.g., left-deep trees [22, 23, 24]).

Our main contributions are the following:

- we formalize a set of relevant operator-placement problems;
- we establish complexity results (all problems turn out to be NP-complete);
- we derive an integer linear programming formulation of the problem;
- we propose several heuristics for the constructive scenario; and
- we compare heuristics through extended simulations, and assess their absolute performance with respect to the optimal solution returned by the linear program.

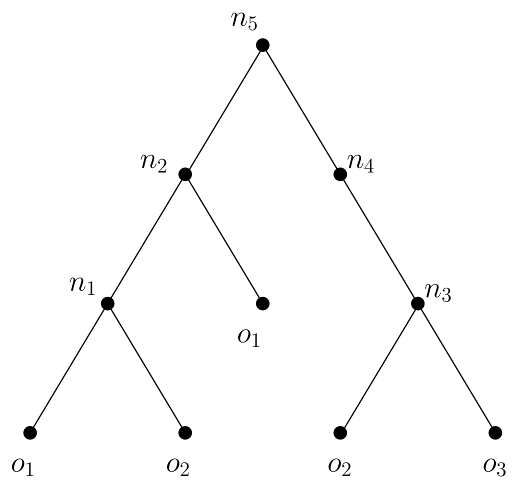
In Section 2 we outline our application and platform models for in-network stream processing. Section 3 defines several relevant resource allocation problems, which are shown to be NP-complete in Section 4. Section 5 derives an integer linear programming formulation of the resource allocation problems. We present several heuristics for solving one of our resource allocation problems in Section 6. These heuristics are evaluated in Section 7. Finally, we conclude the paper in Section 8 with a brief summary of our results and future directions for research.

2 Models

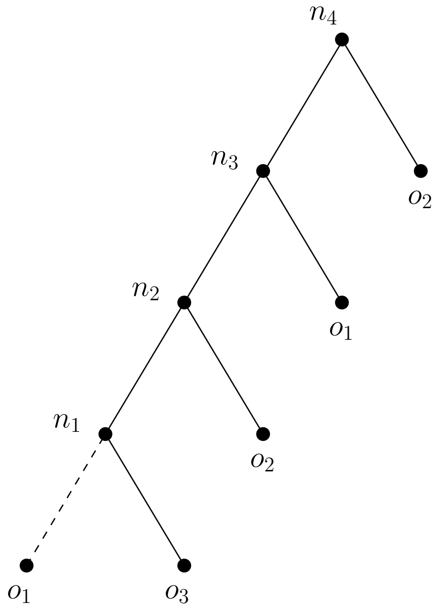
2.1 Application model

We consider an application that can be represented as a set of operators, $\mathcal{N}$. These operators are organized as a binary tree, as shown in Figure 1. Operations are initially performed on basic objects, which are made available and continuously updated at given locations in a distributed network. We denote the set of basic objects $\mathcal{O} = \{o_1, o_2, o_3, \dots\}$. The leaves of the tree are thus the basic objects, and several leaves may correspond to the same object, as illustrated in the figure. Internal nodes (labeled $n_1, n_2, n_3, \dots$) represent operator computations. We call those operators that have at least one basic object as a child in the tree an *al-operator* (for "almost leaf"). For an operator n_i we define:

- *Leaf*(i): the index set of the basic objects that are needed for the computation of n_i, if any;



(a) Standard tree.



(b) Left-deep tree.

Figure 1: Examples of applications structured as a binary tree of operators.

- *Child*(i): the index set of the node's children in $\mathcal{N}$, if any;
- *Parent*(i): the index of the node's parent in $\mathcal{N}$, if it exists.

We have the constraint that $|\mathit{Leaf}(i)| + |\mathit{Child}(i)| \leq 2$ since our tree is binary. All functions above are extended to sets of nodes: $f(I) = \cup_{i \in I} f(i)$, where I is an index set and f is *Leaf*, *Child* or *Parent*.

The application must be executed so that it produces final results, where each result is generated by executing the whole operator tree once, at a target rate. We call this rate the application *throughput* ρ and the specification of the target throughput is a QoS requirement for the application. Each operator $n_i \in \mathcal{N}$ must compute (intermediate) results at a rate at least as high as the target application throughput. Conceptually, a server executing an operator consists of two concurrent threads that run in steady-state:

- One thread periodically downloads the most recent copies of the basic objects corresponding to the operator's leaf children, if any. For our example tree in Figure 1(a), n_1 needs to download o_1 and o_2 while n_2 downloads only o_1 and n_5 does not download any basic object. Note that these downloads may simply amount to constant streaming of data from sources that generate data streams. Each download has a prescribed cost in terms of bandwidth based on application QoS requirements (e.g., so that computations are performed using sufficiently up-to-date data). A basic object o_k has a size δ_k (in bytes) and needs to be downloaded by the processors that use it with frequency f_k. Therefore, these basic object downloads consume an amount of bandwidth equal to $\mathit{rate}_k = \delta_k \times f_k$ on each network link and network card through which this object is communicated.

- Another thread receives data from the operator's non-leaf children, if any, and performs some computation using downloaded basic objects and/or data received from other operators. The operator produces some output that needs to be passed to its parent operator. The computation of operator n_i (to evaluate the operator once) requires w_i operations, and produces an output of size δ_i.

In this paper we sometimes consider *left-deep* trees, i.e., binary trees in which the right child of an operator is always a leaf. These trees arise in practical settings [22, 23, 24] and we show an example of left-deep tree in Figure 1(b). Here $\mathit{Child}(i)$ and $\mathit{Leaf}(i)$ have cardinal 1 for every operator n_i but for the bottom-most operator, n_j, for which $\mathit{Child}(j)$ has cardinal 0, and $\mathit{Leaf}(j)$ has cardinal 1 or 2 depending on the application.

2.2 Platform model

The target distributed network is a fully connected graph (i.e., a clique) interconnecting a set of resources $\mathcal{R} = \mathcal{P} \cup \mathcal{S}$, where $\mathcal{P}$ denotes compute servers, or *processors* for short, and $\mathcal{S}$ denotes data servers, or *servers* for short. Servers hold and update basic objects, while processors apply operators of the application tree. Each server $S_l \in \mathcal{S}$ (resp. processor $P_u \in \mathcal{P}$) is interconnected to the network via a network card with maximum bandwidth Bs_l (resp. Bp_u). The network link from a server S_l to a processor P_u has bandwidth $bs_{l,u}$; on such links the server sends data and the processor receives it. The link between two distinct processors P_u and P_v is bidirectional and it has bandwidth $bp_{u,v}(= bp_{v,u})$ shared by communications in both directions. In addition, each processor $P_u \in \mathcal{P}$ is characterized by a compute speed s_u.

Resources operate under the full-overlap, bounded multi-port model [25]. In this model, a resource R_u can be involved in computing, sending data, and receiving data simultaneously. Note that servers only send data, while processors engage in all three activities. A resource R, which is either a server or a processor, can be connected to multiple network links (since we assume a clique network). The "multi-port" assumption states that R can send/receive data simultaneously on multiple network links. The "bounded" assumption states that the total transfer rate of data sent/received by resource R is bounded by its network card bandwidth (Bs_l for server S_l, or Bp_u for processor P_u).

2.3 Mapping Model and Constraints

Our objective is to map operators, i.e., internal nodes of the application tree, onto processors. As explained in Section 2.1, if a tree node has leaf children it must continuously download up-to-date basic objects, which consumes bandwidth on its processor's network card. Each used processor is in charge of one or several operators. If there is only one operator on processor P_u, while the processor computes for the t-th final result it sends to its parent (if any) the data corresponding to intermediate results for the $(t-1)$-th final result. It also receives data from its non-leaf children (if any) for computing the $(t+1)$-th final result. All three activities are concurrent (see Section 2.2). Note however that different operators can be assigned to the same processor. In this case, the same overlap happens, but possibly on different result instances (an operator may be applied for computing the t_1-th result while another is being applied for computing the t_2-th). The time required by each activity must be summed for all operators to determine the processor's computation time.

We assume that a basic object can be duplicated, and thus be available and updated at multiple servers. We assume that duplication of basic objects is achieved in some out-of-band manner specific to the target application. For instance, this could be achieved via the use of a distributed database infrastructure that enforces consistent data replication. In this case, a processor can choose among multiple data sources when downloading a basic object. Conversely, if two operators have the same basic object as a leaf child and are mapped to different processors, they must both continuously download that object (and incur the corresponding network overheads).

We denote the mapping of the operators in $\mathcal{N}$ onto the processors in $\mathcal{P}$ using an allocation function a: $a(i) = u$ if operator n_i is assigned to processor P_u. Conversely, $\bar{a}(u)$ is the index set of operators mapped on P_u: $\bar{a}(u) = \{i|a(i) = u\}$.

We also introduce new notations to describe the location of basic objects. Processor P_u may need to download some basic objects from some servers. We use $download(u)$ to denote the set of (k, l) couples where object o_k is downloaded by processor P_u from server S_l.

Given these notations we can now express the constraints for the required application throughput, ρ. Essentially, each processor has to communicate and compute fast enough to achieve this throughput, which is expressed via a set of constraints. Note that a communication occurs only when a child or the parent of a given tree node and this node are mapped on different processors. In other terms, we neglect intra-processor communications.

- Each processor P_u cannot exceed its computation capability:

$$\forall P_u \in \mathcal{P}, \quad \sum_{i \in \bar{a}(u)} \rho \cdot \frac{w_i}{s_u} \leq 1 \tag{1}$$

- P_u must have enough bandwidth capacity to perform all its basic object downloads and all communication with other processors. This is expressed by the following constraint, in which the first term corresponds to basic object downloads, the second term corresponds to inter-node communications when a tree node is assigned to P_u and its parent node is assigned to another processor, and the third term corresponds to inter-node communications when a node is assigned to P_u and some of its children nodes are assigned to another processor:
  $\forall P_u \in \mathcal{P}$,

$$\sum_{(k,l) \in download(u)} rate_k + \sum_{j \in Child(\bar{a}(u)) \setminus \bar{a}(u)} \rho.\delta_j + \sum_{j \in Parent(\bar{a}(u)) \setminus \bar{a}(u)} \ \sum_{i \in Child(j) \cap \bar{a}(u)} \rho.\delta_i \leq Bp_u \tag{2}$$

- Server S_l must have enough bandwidth capacity to support all the downloads of the basic objects it holds at their required rates:

$$\forall S_l \in \mathcal{S}, \quad \sum_{P_u \in \mathcal{P}} \ \sum_{(k,l) \in download(u)} rate_k \leq Bs_l \tag{3}$$

- The link between server S_l and processor P_u must have enough bandwidth capacity to support all possible object downloads from S_l to P_u at the required rate:

$$\forall P_u \in \mathcal{P}, \forall S_l \in \mathcal{S}, \quad \sum_{(k,l) \in download(u)} rate_k \leq bs_{l,u} \tag{4}$$

- The link between processor P_u and processor P_v must have enough bandwidth capacity to support all possible communications between the tree nodes mapped on both processors. This constraint can be written similarly to constraint (2) above, but without the cost of basic object downloads, and with specifying that P_u communicates with P_v:

$$\forall P_u, P_v \in \mathcal{P}, \quad \sum_{j \in Child(\bar{a}(u)) \cap \bar{a}(v)} \rho.\delta_j + \sum_{j \in Parent(\bar{a}(u)) \cap \bar{a}(v)} \sum_{i \in Child(j) \cap \bar{a}(u)} \rho.\delta_i \leq bp_{u,v} \quad (5)$$

3 Problem Definitions

The overall objective of the operator-mapping problem is to ensure that a prescribed throughput is achieved while minimizing a cost function. We consider two broad cases. In the first case, the user must buy processors (with various computing speed and network card bandwidth specifications) and build the distributed network dedicated to the application. For this "constructive" problem, which we call Constr, the cost function is simply the actual monetary cost of the purchased processors. This problem is relevant to, for instance, the surveillance application mentioned in Section 1. The second case, which we call Non-Constr, targets an existing platform. The goal is then to use a subset of this platform so that the prescribed throughput is achieved while minimizing a cost function. Several cost functions can be envisioned, including the compute capacity or the bandwidth capacity used by the application in steady state, or a combination of the two. In the following, we consider a cost function that accounts solely for processors. This function be based on a processor's processing speed and on the bandwidth of its network card.

Different platform types may be considered for both the Constr and the Non-Constr problems depending on the heterogeneity of the resources. In the Constr case, we assume that some standard interconnect technology is used to connect all the processors together ($bp_{u,v} = bp$). We also assume that the same interconnect technology is used to connect each server to processors ($bs_{l,u} = bs_l$). We consider the case in which the processors are homogeneous because only one type of CPUs and network cards can be purchased ($Bp_u = Bp$ and $s_u = s$). We term the corresponding problem Constr-Hom. We also consider the case in which the processors are heterogeneous with various compute speeds and network card bandwidth, which we term Constr-LAN. In the Non-Constr case we consider the case in which the platform is fully homogeneous, which we term Non-Constr-Hom. We then consider the case in which the processors are heterogeneous but the network links are homogeneous ($bp_{u,v} = bp$ and $bs_{l,u} = bs_l$), which we term Non-Constr-LAN. Finally we consider the fully heterogeneous case in which network links can have various bandwidths, which we term Non-Constr-Het.

Homogeneity in the platform as described above applies only to processors and not to servers. Servers are always fixed for a given application, together with the objects they hold. We sometimes consider variants of the problem in which the servers and application tree have particular characteristics. We denote by HomS the case then all servers have identical network capability ($Bs_l = Bs$) and communication links to processors ($bs_{l,u} = bs$). We can also consider the mapping of particular trees, such as left-deep trees (LDTree) and/or homogeneous trees with identical object rates $rate_k = rate$ and computing costs $w_i = w$ (HomA). Also, we can consider application trees with no communication cost ($\delta_i = 0$, NoComA). All these variants correspond to simplifications of the problem, and we simply append HomS, LDTree, HomA, and/or NoComA to the problem name to denote these simplifications.

4 Complexity

Without surprise, most problem instances are NP-hard, because downloading objects with different rates on two identical servers is the same problem as 2-Partition [26]. But from a theoretical point of view, it is important to assess the complexity of the simplest instance of the problem, i.e., mapping a fully homogeneous left-deep tree application with objects placed on a fully homogeneous set of servers, onto a fully homogeneous set of processors: Constr-Hom-HomS-LDTree-HomA-NoComA (or C-LDT-Hom for short). It turns out that even this problem is difficult, due to the combinatorial space induced by the mapping of basic objects that are shared by several operators. Note that the corresponding non-constructive problem is exactly the same, since it aims at minimizing the number of selected processors given a pool of identical processors. This complexity result thus holds for both classes of problems.

Definition 1. *The problem* C-LDT-Hom *(*Constr-Hom-HomS-LDTree-HomA-NoComA*) consists in minimizing the number of processors used in the application execution. K is the prescribed throughput that should not be violated.* C-LDT-Hom-Dec *is the associated decision problem: given a number of processors N, is there a mapping that achieves throughput K?*

Theorem 1. C-LDT-Hom-Dec *is NP-complete.*

Proof. First, C-LDT-Hom-Dec belongs to NP. Given an allocation of operators to processors and the download list $download(u)$ for each processor P_u, we can check in polynomial time that we use no more than N processors, that the throughput of each enrolled processor respects K:

$$K \times |\bar{a}(u)| \, \frac{w}{s} \leq 1 \, ,$$

and that bandwidth constraints are respected.

To establish the completeness, we use a reduction from 3-Partition, which is NP-complete in the strong sense [26]. We consider an arbitrary instance $\mathfrak{I}_1$ of 3-Partition: given $3n$ positive integer numbers $\{a_1, a_2, \ldots, a_{3n}\}$ and a bound R, assuming that $\frac{R}{4} < a_i < \frac{R}{2}$ for all i and that $\sum_{i=1}^{3n} a_i = nR$, is there a partition of these numbers into n subsets $I_1, I_2, \ldots, I_n$ of sum R? In other words, are there n subsets $I_1, I_2, \ldots, I_n$ such that $I_1 \cup I_2 \ldots \cup I_n = \{1, 2, \ldots, 3n\}$, $I_i \cap I_j = \emptyset$ if $i \neq j$, and $\sum_{j \in I_i} a_j = R$ for all i (and $|I_i| = 3$ for all i). Because 3-Partition is NP-complete in the strong sense, we can encode the $3n$ numbers in unary and assume that the size of $\mathfrak{I}_1$ is $O(n + M)$, where $M = \max_i\{a_i\}$.

We build the following instance $\mathfrak{I}_2$ of C-LDT-Hom-Dec :

- The object set is $\mathcal{O} = \{o_1, ..., o_{3n}\}$, and there are $3n$ servers each holding an object, thus o_i is available on server S_i. The rate of o_i is $rate = 1$, and the bandwidth limit of the servers is set to $Bs = 1$.

- The left-deep tree consists of $|\mathcal{N}| = nR$ operators with $w = 1$. Each object o_i appears a_i times in the tree (the exact location does not matter), so that there are $|\mathcal{N}|$ leaves in the tree, each associated to a single operator of the tree.

- The platform consists of n processors of speed $s = 1$ and bandwidth $Bp = 3$. All the link bandwidths interconnecting servers and processors are equal to $bs = bp = 1$.

- Finally we ask whether there exists a solution matching the bounds $1/K = R$ and $N = n$.

The size of $\mathfrak{I}_2$ is clearly polynomial in the size of $\mathfrak{I}_1$, since the size of the tree is bounded by $3nM$. We now show that instance $\mathfrak{I}_1$ has a solution if and only if instance $\mathfrak{I}_2$ does.

Suppose first that $\mathfrak{I}_1$ has a solution. We map all operators corresponding to occurrences of object o_j, $j \in I_i$, onto processor P_i. Each processor receives three distinct objects, each coming from a different server, hence bandwidths constraints are satisfied. Moreover, the number of operators computed by P_i is equal to $\sum_{j \in I_i} a_i = R$, and the required throughput it achieved because $KR \leq 1$. We have thus built a solution to $\mathfrak{I}_2$.

Suppose now that $\mathfrak{I}_2$ has a solution, i.e., a mapping matching the bound $1/K = R$ with n processors. Due to bandwidth constraints, each of the n processors is assigned at most three distinct objects. Conversely, each object must be assigned to at least one processor and there are $3n$ objects, so each processor is assigned exactly 3 objects in the solution, and no object is sent to two distinct processors. Hence, a processor must compute all operators corresponding to the objects it needs to download, which directly leads to a solution of $\mathfrak{I}_1$ and concludes the proof. □

Note that problem C-LDT-Hom-Dec becomes polynomial if one adds the additional restriction that no basic object is used by more than one operator in the tree. In this case, one can simply assign operators to $\lceil |\mathcal{N}| \times w/s \rceil$ arbitrary processors in a round-robin fashion.

5 Linear Programming Formulations

In this section, we formulate the Constr optimization problem as an integer linear program (ILP). We deal with the most general instance of the problem Constr-LAN. Then we explain how to transform this integer linear program to formulate the Non-Constr-Het problem.

5.1 ILP for Constr

Constants – We first define the set of constant values that define our problem. The application tree is defined via parameters par and $leaf$, and the location of objects on servers is defined via parameter obj. Other parameters are defined with the same notations as previously introduced: δ_i, w_i for operators, $rate_k$ for object download rates, and Bs_l for server network card bandwidths. More formally:

- $par(i, j)$ is a boolean variable equal to 1 if operator n_i is the parent of n_j in the application tree, and 0 otherwise.
- $leaf(i, k)$ is a boolean variable equal to 1 if operator n_i requires object o_k for computation, i.e., o_k is a children of n_i in the tree. Otherwise $leaf(i, k) = 0$.
- $obj(k, l)$ is a boolean variable equal to 1 if server S_l holds a copy of object o_k.
- $\delta_i, w_i, rate_k, Bs_l$ are rational numbers.

The platform can be built using different types of processors. More formally, we consider a set $\mathcal{C}$ of processor specifications, which we call "classes". We can acquire as many processors of a class $c \in \mathcal{C}$ as needed, although no more than $\mathcal{N}$ processors are necessary overall. We denote the cost of a processor in class c by $cost_c$. Each processor of class c has computing speed s_c and network card bandwidth Bp_c. The link bandwidth between processors is a constant bp, while the link between a server S_l and a processor is bs_l. For each class, processors are numbered from 1 to $|\mathcal{N}|$, and $P_{c,u}$ refers to the u^{th} processor of class c. Finally, ρ is the throughput that must be achieved by the application:

- $cost_c, s_c, Bp_c, bp, bs_l$ are rational numbers;
- ρ is a rational number.

Variables – Now that we have defined the constants that define our problem we define unknown variables to be computed:

- $x_{i,c,u}$ is a boolean variable equal to 1 if operator n_i is mapped on $P_{c,u}$, and 0 otherwise. There are $|\mathcal{N}|^2.|\mathcal{C}|$ such variables, where $|\mathcal{C}|$ is the number of different classes of processors.
- $d_{c,u,k,l}$ is a boolean variable equal to 1 if processor $P_{c,u}$ downloads object o_k from server S_l, and 0 otherwise. The number of such variables is $|\mathcal{C}|.|\mathcal{N}|.|\mathcal{O}|.|\mathcal{S}|$.
- $y_{i,c,u,i',c',u'}$ is a boolean variable equal to 1 if n_i is mapped on $P_{c,u}$, $n_{i'}$ is mapped on $P_{c',u'}$, and n_i is the parent of $n_{i'}$ in the application tree. There are $|\mathcal{N}|^4.|\mathcal{C}|^2$ such variables.
- $used_{c,u}$ is a boolean variable equal to 1 if processor $P_{c,u}$ is used in the final mapping, i.e., there is at least one operator mapped on this processor, and 0 otherwise. There are $|\mathcal{C}|.|\mathcal{N}|$ such variables.

Constraints – Finally, we must write all constraints involving our constants and variables. In the following, unless stated otherwise, i, i', u and u' span set $\mathcal{N}$; c and c' span set $\mathcal{C}$; k spans set $\mathcal{O}$; and l spans set $\mathcal{S}$. First we need constraints to guarantee that the allocation of operators to processors is a valid allocation, and that all required downloads of objects are done from a server that holds the corresponding object.

- $\forall i \;\; \sum_{c,u} x_{i,c,u} = 1$: each operator is placed on exactly one processor;
- $\forall c, u, k, l \;\; d_{c,u,k,l} \le obj(k,l)$: object o_k can be downloaded from S_l only if S_l holds o_k;
- $\forall c, u, k, l \;\; d_{c,u,k,l} \le \sum_i x_{i,c,u}.leaf(i,k)$: if there is no operaotr assigned to $P_{c,u}$ that requires object k, then $P_{c,u}$ does not need to download object k and $d_{c,u,k,l} = 0$ for all server S_l.
- $\forall i, k, c, u \;\; 1 \ge \sum_l d_{c,u,k,l} \ge x_{i,c,u}.leaf(i,k)$: processor $P_{c,u}$ must download object o_k from exactly one server if there is an operator n_i mapped on this processor that requires o_k for computation.

The next set of constraints aim at properly constraining variable $y_{i,c,u,i',c',u'}$. Note that a straightforward definition would be $y_{i,c,u,i',c',u'} = par(i,j).x_{i,c,u}.x_{i',c',u'}$, i.e., a logical conjunction between three conditions. Unfortunately, this definition makes our program non-linear as two of the conditions are variables. Instead, for all i, c, u, i', c', u', we write:

- $y_{i,c,u,i',c',u'} \leq par(i,j)$; $y_{i,c,u,i',c',u'} \leq x_{i,c,u}$; $y_{i,c,u,i',c',u'} \leq x_{i',c',u'}$: y is forced to 0 if one of the conditions does not hold.

- $y_{i,c,u,i',c',u'} \geq par(i,j).\left(x_{i,c,u} + x_{i',c',u'} - 1\right)$: y is forced to be 1 only if the three conditions are true (otherwise the right term is less than or equal to 0).

The following constraints ensure that $used_{c,u}$ is properly defined:

- $\forall c, u\ used_{c,u} \leq \sum_i x_{i,c,u}$: processor $P_{c,u}$ is not used if no operator is mapped on it;

- $\forall c, u, i\ used_{c,u} \geq x_{i,c,u}$: processor $P_{c,u}$ is used if at least one operator n_i is mapped to it.

Finally, we have to ensure that the required throughput is achieved and that the various bandwidth capacities are not exceeded, following equations (1)-(5).

- $\forall c, u\ \ \sum_i x_{i,c,u}.\rho\frac{w_i}{s_c} \leq 1$: the computation of each processor must be fast enough so that the throughput is at least equal to ρ;

- $\forall c, u\ \ \sum_{k,l} d_{c,u,k,l}.rate_k + \sum_{i,i',(c',u')\neq(c,u)} y_{i,c,u,i',c',u'}.\rho.\delta_{i'} + \sum_{i,i',(c',u')\neq(c,u)} y_{i',c',u',i,c,u}.\rho.\delta_i \leq Bp_c$: bandwidth constraint for the processor network cards;

- $\forall l\ \ \sum_{c,u,k} d_{c,u,k,l}.rate_k \leq Bs_l$: bandwidth constraint for the server network cards;

- $\forall l, c, u\ \ \sum_k d_{c,u,k,l}.rate_k \leq bs_l$: bandwidth constraint for links between servers and processors;

- $\forall c, u, c', u'$ with $(c,u) \neq (c',u')$ $\sum_{i,i'} y_{i,c,u,i',c',u'}.\rho.\delta_{i'} + \sum_{i,i'} y_{i',c',u',i,c,u}.\rho.\delta_i \leq bp$: bandwidth constraint for links between processors.

Objective function.
We aim at minimizing the cost of used processors, thus the objective function is

$$\min\left(\sum_{c,u} used_{c,u}.cost_c\right).$$

5.2 ILP for Non-Constr

The linear program for the Non-Constr problem is very similar to the Constr one, except that the platform is known a-priori. Furthermore, we no longer consider processor classes. However, we can simply assume that there is only one processor of each class, and define $|\mathcal{C}| = |\mathcal{P}|$, the set of processors of the platform. The *number* of processors of class c is then limited to 1. As a result, all indices u in the previous linear program are removed, and we obtain a linear program formulation of the Non-Constr-LAN problem. The number of variables and constraints is reduced from $|\mathcal{N}|$ to 1 when appropriate. We can further generalize the linear program to Non-Constr-Het, by adding links of different bandwidths between processors. We just need to replace bp by $bp_{c,c'}$ and bs_l by $bs_{l,c}$ every time they appear in the linear program in the previous section. Altogether, we have provided integer linear program formulations for all our constructive and non-constructive problems.

6 Heuristics

In this section we propose several heuristics to solve the Constr operator-placement problem. Due to lack of space, we leave the development of heuristics for the Non-Constr problem outside the scope of this paper. We choose to focus on constructive scenarios because such scenarios are relevant to practice and, to the best of our knowledge, have not been studied extensively in the literature. We say that the heuristics can then "purchase" processors, or "sell back" processors, until a final set of needed processors is determined.

We consider two types of heuristics: (i) operator placement heuristics and (ii) object download heuristics. In a first step, an operator placement heuristic is used to determine the number of processors that should be purchased, and to decide which operators are assigned to which processors. Note that all our heuristics fail if a single operator cannot be treated by the most expensive processor with the desired throughput. In a second step, an object download heuristic is used to decide from which server each processor downloads the basic objects that are needed for the operators assigned to this processor. In the next two sections we propose several candidate heuristics both for operator placement and object download.

6.1 Operator Placement Heuristics

6.1.1 Random

While there are some unassigned operators, the Random heuristic picks one of these unassigned operators randomly, called *op*. It then purchases the cheapest possible processor that is able to handle *op* while achieving the required application throughput. If there is no such processor, then the heuristic considers *op* along with one of its children operators or with its parent operator. This second operator is chosen so that it has the most demanding communication requirements with *op* (the intuition is that we try to reduce communication overhead). If no processor can be acquired that can handle both the operators together, then the heuristic fails. If the additional operator had already been assigned to another processor, this last processor is sold back.

6.1.2 Comp-Greedy

The Comp-Greedy heuristic first sorts operators in non-increasing order of w_i, i.e., most computationally demanding operators first. While there are unassigned operators, the heuristic purchases the most expensive processor available and assigns the most computationally demanding unassigned operator to it. If this operator cannot be processed on this processor so that the required throughput is achieved, then the heuristic uses a grouping technique similar to that used by the Random heuristic (i.e., trying to group the operator with its child or parent operator with which it has the most demanding communication requirement). If after this step some capacity is left on the processor, then the heuristic tries to assign other operators to it. These operators are picked in non-increasing order of w_i, i.e., trying to first assign to this processor the most computationally demanding operators. Once no more operators can be assigned to the processor, the heuristic attempts to "downgrade" the processor. This downgrading consists in, if possible, replacing the current processor by the cheapest processor available that can still handle all the operators assigned on the current processor.

6.1.3 Comm-Greedy

The Comm-Greedy heuristic attempts to group operators to reduce communication costs. It picks the two operators that have the largest communication requirements. These two operators are grouped and assigned to the same processor, thus saving the costly communication between both processors. There are three cases to consider for this assignment: (i) both operators were unassigned, in which case the heuristic simply purchases the cheapest processor that can handle both operators; if no such processor is available then the heuristic purchases the most expensive processor for each operator; (ii) one of the operators was already assigned to a processor, in which case the heuristic attempts to accommodate the other operator as well; if this is not possible then the heuristic purchases the most expensive processor for the other operator; (iii) both operators were already assigned on two different processors, in which case the heuristic attempts to accommodate both operators on one processor and sell the other processor; if this is not possible then the current operator assignment is not changed.

6.1.4 Object-Greedy

The Object-Greedy heuristic attempts to group operators that need the same basic objects. Recall that an al-operator is an operator that requires at least one basic object. The heuristic sorts all al-operators by the maximum required download frequency of the basic objects they require, i.e., in non-increasing order of maximum $rate_j$ values (and w_i in case of equality). The heuristic then purchases the most expensive processor and assigns the first such operators to it. Once again, if the most expensive processor cannot handle this operator, the heuristic attempts to group the operator with one of its unassigned parent or child operators. If this is not possible, then the heuristic fails. Then, in a greedy fashion, this processor is filled first with al-operators and then with other operators as much as possible.

6.1.5 Subtree-Bottom-Up

The Subtree-Bottom-Up heuristic first purchases as many most expensive processors as there are al-operators and assigns each al-operator to a distinct processor. The heuristic then tries to merge the operators with their father on a single machine, in a bottom-up fashion (possibly leading to the selling back of some processors). Consider a processor on which a number of operators have been assigned. The heuristic first tries to allocate as many parent operators of the currently assigned operators to this processor. If some parent operators cannot be assigned to this processor, then one or more new processors are purchased. This mechanism is used until all operators have been assigned to processors.

6.1.6 Object-Grouping

For each basic object, this heuristic counts how many operators need this basic object. This count is called the “popularity” of the basic object. The al-operators are then sorted by non-increasing sum of the popularities of the basic object they need. The heuristic starts by purchasing the most expensive processor and assigning to it the first al-operator. The heuristic then attempts to assign as many other al-operators that require the same basic objects as the first al-operator, taken in order of non-increasing popularity, and then as many non al-operators as possible. This process is repeated until all operators have been assigned.

6.1.7 Object-Availability

This heuristic takes into account the distribution of basic objects on the servers. For each object k the number av_k of servers handling object o_k is calculated. Al-operators in turn are treated in increasing order of av_k of the basic objects they need to download. The heuristic tries to assign as many al-operators downloading object k as possible on a most expensive processor. The remaining internal operators are assigned in the same mechanism as Comp-Greedy proceeds, i.e., in decreasing order of w_i of the operators.

6.2 Object Download Heuristics

Once an operator placement heuristic has been executed, each al-operator is mapped on a processor, which needs to download basic objects required by the operator. Thus, we need to specify from which server this download should occur. Two server selection heuristics are proposed in order to define, for each processor, the server from which required basic objects are downloaded.

6.2.1 Server-Selection-Random

This heuristic is only used in combination with Random. Once Random has decided about the mapping of operators onto processors, Server-Selection-Random associates randomly a server to each basic object a processor has to download.

6.2.2 Server-Selection-Intelligent

This server selection heuristic is more sophisticated and is used in combination with all operator placement heuristics except Random. Server-Selection-Intelligent uses three loops: the first loop assigns objects that are held in exclusivity, i.e., objects that have to be downloaded from a specific server. If not all downloads can be guaranteed, the heuristic fails. The second loop associates as many downloads as possible to servers that provide only one basic object type. The last loop finally tries to assign the remaining basic objects that have to be downloaded. For this purpose objects are treated in decreasing order of interestedProcs/numPossibleServers, where interestedProcs is the remaining number of processors that need to download the object and numPossibleServers is the number of servers where the object still can be downloaded. In the decision process servers are considered in decreasing order of min(remainingBW, linkBW), where remainingBW is the remaining capacity of the servers network card and linkBW is the bandwidth of the communication link.

Once the server association process is done, a processor downgrade procedure is called. All processors are replaced by the less expensive model that fulfills the CPU and network card requirements of the allocation.

7 Simulation Results

7.1 Resource Cost Model

In order to instantiate our simulations with realistic models for resource costs, we use information available from the Dell Inc. Web site. More specifically, we use the prices for configurations of Intel's latest, high-end, rack-mountable server (PowerEdge R900), as advertised

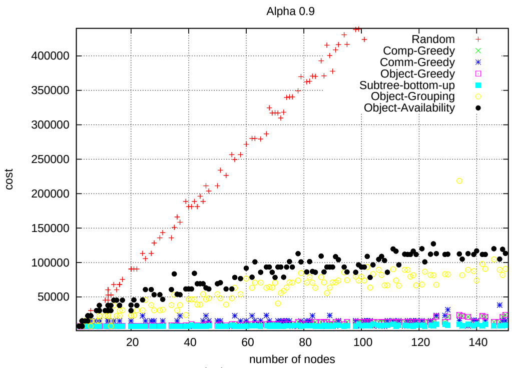
Table 1: Incremental costs for increases in processor performance or network card bandwidth relative to a $7,548 base configuration (based on data from the Dell Inc. web site, as of early March 2008).

Processor			Network Card		
Performance (GHz)	Cost ($)	Ratio (GHz/$)	Bandwidth (Gbps)	Cost ($)	Ratio (Gbps/$)
11.72	7,548 + 0	1.55×10^{-3}	1	7,548 + 0	1.32×10^{-4}
19.20	7,548 + 1,550	1.93×10^{-3}	2	7,548 + 399	2.51×10^{-4}
25.60	7,548 + 2,399	2.38×10^{-3}	4	7,548 + 1,197	4.57×10^{-4}
38.40	7,548 + 3,949	3.12×10^{-3}	10	7,548 + 2,800	9.66×10^{-4}
46.88	7,548 + 5,299	3.43×10^{-3}	20	7,548 + 5,999	14.76×10^{-4}

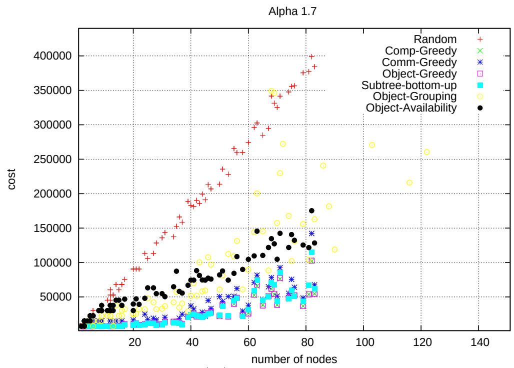
on the Web site as of early March 2008. Due to the large number of available configurations, we only consider processor cores with 8MB L1 caches (so that their performances are more directly comparable), and with optical Gigabit Ethernet (GbE) network cards manufactured by Intel Inc. For simplicity, we assume that the effective bandwidth of a network card is equal to its peak performance. In reality, we know that, say, a 10GbE network card delivers a bandwidth lower than 10Gbps due to various software and hardware overheads. We also make the assumption that the performance of a multi-processor multi-core server is proportional to the sum of the clock rates of all its cores. This assumption generally does not hold in practice due, e.g., to parallelization overhead and cache sharing. It is outside the scope of this work to develop (likely elusive) generic performance models for network cards and multi-processor multi-core servers, but we argue that the above assumptions still lead to a reasonable resource cost model. The configuration prices are show in Table 1, relative to the base configuration, whose cost is $7,548. Note that we do not consider configurations designed for low power consumption, which achieve possibly lower performance at higher costs.

7.2 Simulation Methodology

All our simulations use randomly generated binary operator trees with at most N operators, which can be specified. All leaves correspond to basic objects, and each basic object is chosen randomly among 15 different types. For each of these 15 basic object types, we randomly choose a fixed size. In simulations with *small objects*, the object sizes are in the range 5-30MB, whereas *big objects* have data sizes in the range 450-530MB. The download frequency for basic objects is either fixed to 1/50s or 1/2s. The computation amount w_n for an operator n (a non-leaf node in the tree), depends on its children l and r: $w_n = (\delta_l + \delta_r)^\alpha$, where α is a constant fixed for each simulation run. The same principle is used for the output size of each operator, using a constant $\beta = 1.0$ for all simulations. The application throughput ρ is fixed to 1.0 for all simulations. Throughout the whole set of simulations we use the same server architecture: we dispose of 6 servers, each of them is equipped with a 10 GB network card. Objects of our 15 types are randomly distributed over the 6 servers. We assume that servers and processors are all interconnected by a 1GB link. The mapping operator problem is defined by many parameters, an we argue that our simulation methodology, in which several parameters are fixed, is sufficient to compare our various heuristics.



(a) $\alpha = 0.9$.



(b) $\alpha = 1.7$.

Figure 2: Simulation with small basic objects and big download rates, increasing number of operators.

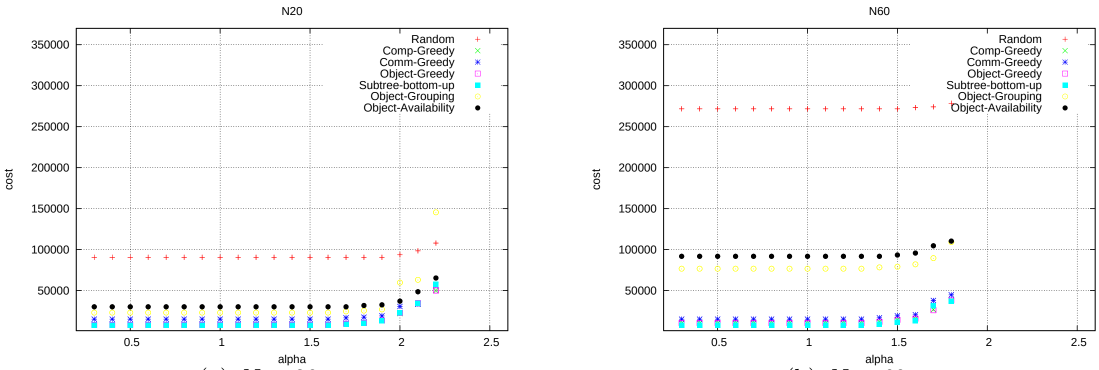
7.3 Results

We present hereafter results for several sets of experiments. Due to lack of space we will only present the most significant figures, but the entire set of figures can be found on the web [27].

High download rates - small object sizes In a first set of simulations, we study the behavior of the heuristics when download rates are high and object sizes small (5-30MB). Figure 2 shows the results, when the number of nodes N in the tree varies, but the computation factor α is fixed. As expected, Random performs poorly and the platform chosen for an application with around 100 operators or more exceeds a cost of \$400,000 (cf. Figure 2(a)), when $\alpha = 0.5$). Subtree-bottom-up achieves the best costs, and for an application with 100 operators it finds a platform for the price of \$8,745. All Greedy heuristics exhibit similar performance, slightly poorer than Subtree-bottom-up, but still withing acceptable costs under \$50,000. Perhaps surprisingly, the heuristics that pay special attention for basic objects, Object-Grouping and Object-Availability, perform poorly.

With a larger value of α (cf. Figure 2(b)) the operator tree size becomes a more limiting factor. For trees with more than 80 operators, almost no feasible mapping can be found. However, the relative performance of our heuristics remains almost the same, with two notable features: a) Object-Grouping still finds some mappings for operator trees bigger up to 120 operators, with costs between \$200,000 and \$275,000; b) Comp-Greedy and Object-Greedy perform as well at at times better than Subtree-bottom-up when the number of operator increases.

Figure 3 shows the comparison of the heuristics when N is fixed and the computation factor α increases. This experiment uses the same parameters as the previous one. Up to a threshold the α parameter has no influence on the heuristics' performance and the solution cost is linear. When α reaches the threshold, the solution cost of each heuristic increases until α exceeds a second threshold and no solution can be found anymore. Depending on the number of operators both thresholds have lower or higher values. In the case of small operator trees with only 20 nodes (see Figure 3(a)), the first threshold is for $\alpha = 1.7$ and the second at $\alpha = 2.2$ (vs. $\alpha = 1.6$ and $\alpha = 1.8$ for operator trees of size 60, as seen in Figure 3(b)). Subtree-bottom-up behaves in both cases the best, whereas Random performs



(a) $N = 20$. (b) $N = 60$.

Figure 3: Simulation with small basic objects and big download rates, increasing α.

the poorest. Object-Grouping and Object-Availability change their position in the ranking: for small trees Object-Grouping behaves better, while for bigger trees it is outperformed by Object-Availability. The Greedy heuristics are between Subtree-bottom-up and the object sensitive heuristics. When α is larger, they at times outperform Subtree-bottom-up.

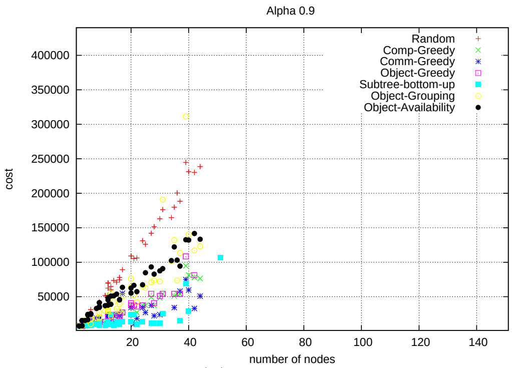
High download rates - big object sizes The second set of experiments analyzes the heuristics' performance under high download rates and big object sizes (450-530MB). As for small object sizes, we plot two types of figures. Figure 4 shows results for a fixed α and increasing number of operators. We see that for trees bigger than 45 nodes, almost no feasible solution can be found, both for α smaller than 1 and higher than 1. In general, Subtree-bottom-up still achieves the best costs, but at times it is outperformed by Comm-Greedy. Subtree-bottom-up even fails in two cases in which other heuristics find a solution: see Figure 4(a), N=41 and N=42. This behavior can be explained as follows. The Subtree-bottom-up routine achieves the best result in terms of processors that have to be purchased. But unfortunately this operator-processor-mapping fails during the server allocation process. (Often the bandwidth of 1 GB between processor and server is not sufficient).

Comm-Greedy achieves in this experiment the best costs among the Greedy heuristics, whereas Random, Object-Availability and Object-Grouping still perform the poorest.

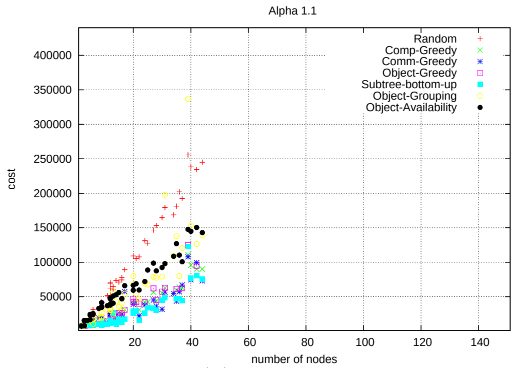
When N is fixed we observe a behavior similar as that for small object sizes. The ranking (Subtree-bottom-up, Greedy, object sensitive, and finally Random) remains unchanged. When $N = 20$, Comp-Greedy outperforms Object-Greedy and Comm-Greedy finds a feasible solution only once (see Figure 5(a)). Object-Availability achieves better results than Object-Grouping.

In the case of $N = 40$ (see Figure 5(b)), the ranking is unchanged but for the fact that Object-Availability and Object-Grouping are swapped. Also, in this case, Object-Greedy never succeeds to find a feasible solution, whereas Comm-Greedy achieves the second best results.

Note that the failure of Object-Greedy depends on the tree structure, and thus our results do not mean that Object-Greedy fails for all tree sizes higher than 20. Here once again, the solution found by the heuristic for the operator mapping leads to the failure in the server association process.

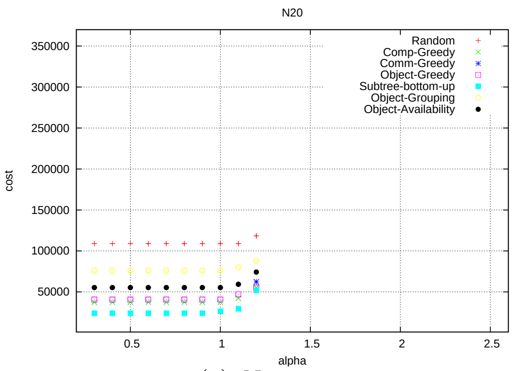


(a) $\alpha = 0.9$.

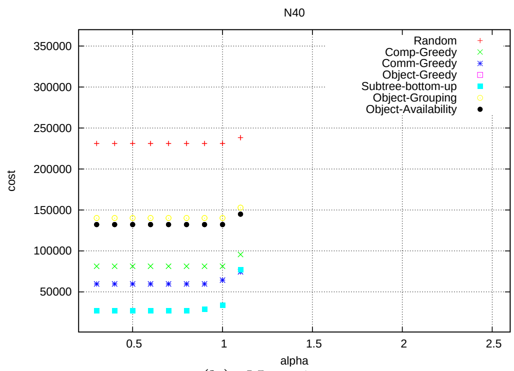


(b) $\alpha = 1.1$.

Figure 4: Simulation with big basic objects and high download rates, increasing number of operators.



(a) $N = 20$.

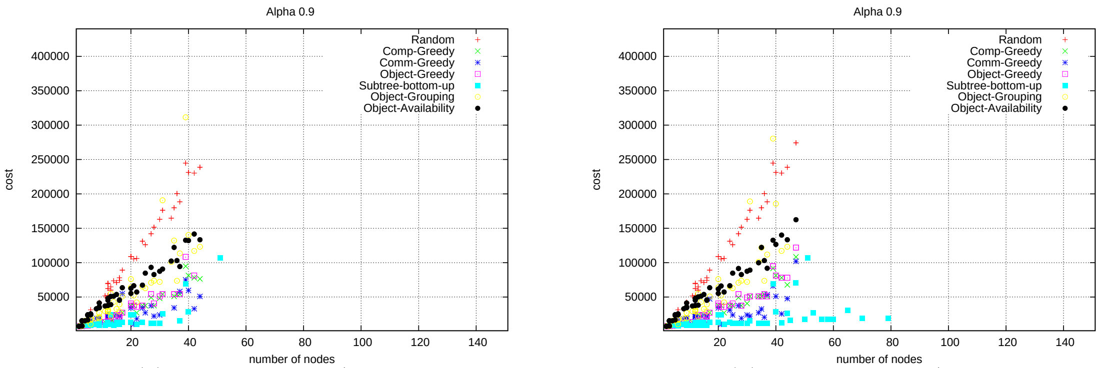


(b) $N = 40$.

Figure 5: Simulation with big basic objects and high download rates, increasing α.

Table 2: Influence of the download rate on the platform cost, in \$, when object sizes are small.

	small object sizes			big object sizes		
N	Comm-Greedy	Obj-Greedy	Subtree-b-up	Comm-Greedy	Obj-Greedy	Subtree-b-up
115	**7947**	13547	8745	**7548**	13547	8745
116	**15495**	13547	**7947**	**15096**	13547	**7548**
117	**7947**	13547	**7947**	**7548**	13547	**7548**
118	**15495**	13547	7548	**15096**	13547	7548
119	**15495**	13547	8745	**15096**	13547	8745



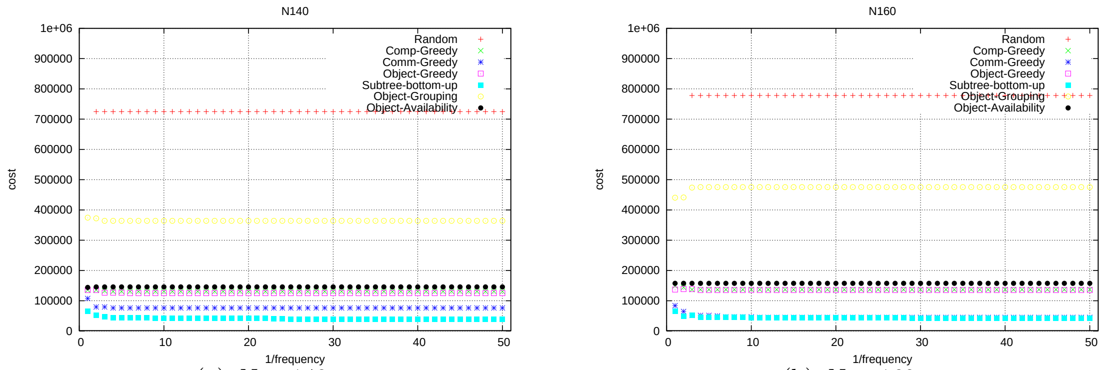
(a) $frequency = 1/50s$. (b) $frequency = 1/2s$.

Figure 6: Comparison between the simulations with big basic objects and different download rates, $\alpha = 0.9$.

Low download rates - small object sizes The behavior of the heuristics when download rates are low, i.e., frequency $= 1/50s$, is almost the same as for high download rates. In general the heuristics lead to the same operator mapping, but in some cases the purchased processors have less powerful network cards (Cf. Table 2).

Low download rates - big object sizes In this case, low download rates slightly improve the success rate of the heuristics (see Figure 6). Indeed, because of the lower download rates the links between servers and processors are less congested, and hence the server association is feasible in more scenarios.

Influence of download rates (frequency) on the solution The third set of experiments studies the influence of download rates on the solution. Remember the download rate of a basic object k is computed by $rate_k = frequency \times \delta_k$. A first results is that frequencies smaller than $1/10s$ has no further influence on the solution. All heuristics will find the same solutions for a fixed operator tree, as seen in Figure 7. For frequencies between $1/2s$ and $1/10s$, the solution cost changes. In general the cost decreases, but for $N = 160$ the cost for the Object-Grouping heuristic increases. Furthermore, the heuristic ranking remains: Subtree-bottom-up, followed by the Greedy family, followed by the object sensitive ones, and Random



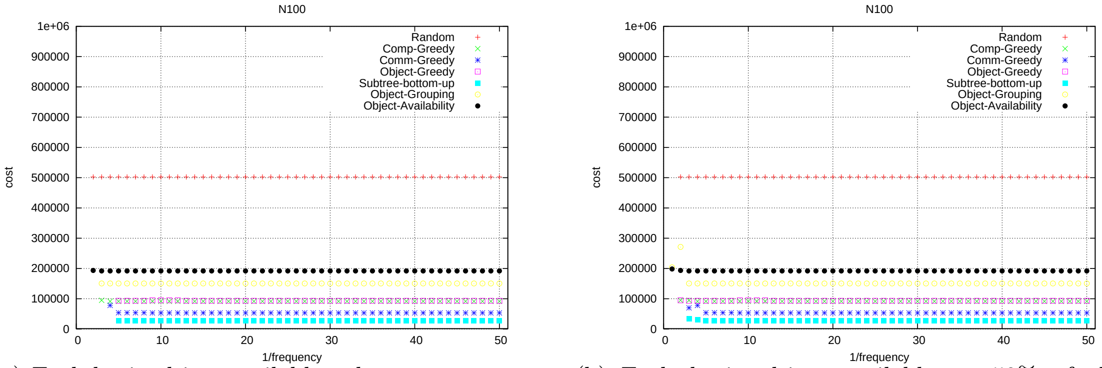
(a) $N = 140$. (b) $N = 160$.

Figure 7: Simulation to evaluate the influence of the frequency that determines download rates.

forms the bottom of the league. Interestingly, the costs of Object-Availability decrease with the number of operators. In this case the number of operators that need to download a basic object increases, and hence the privileged treatment of basic objects in order of availability on servers becomes more important (compare Figure 7 and Figure 8(a)).

We also tested the importance of the number of basic object replications on the servers. Initially we ran experiments also on different server configurations, with basic objects either not replicated are replicated on all servers. However, we did not observe a significant difference in the results across different server configurations. We thus present results only for are default server configuration. Figure 7 shows results for decreasing frequencies, when each basic object is available only on a single server. Comparing this plot to Figure 8(b), for which each basic object is available on 50% of the servers, one notices no significant difference. Focusing solely on frequencies between $1/2s$ and $1/10s$, we see that Subtree-bottom-up, Comm-Greedy, Object-Grouping, and Object-Greedy find more solutions, at frequencies for which they failed before (Figure 8(a)). We conclude that the level of replication of basic objects on servers may matter for application trees with specific structures and download frequencies, but that in general we can consider that this parameter has little or no effect on the performance of the heuristics.

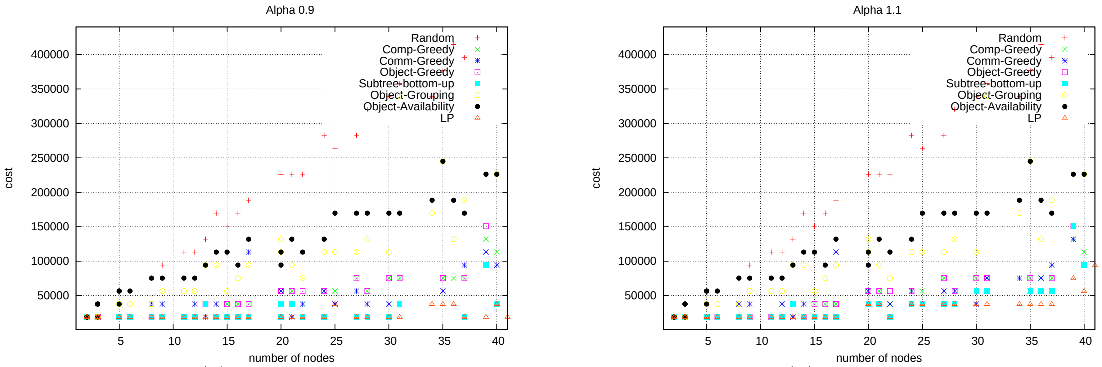
Comparison of the heuristics to a LP solution on a homogeneous platform This last set of experiments is dedicated to the evaluation of our heuristics via a lower bound given by the solution of our integer linear program. We use Cplex 11 to solve our linear program. Unfortunately, the LP is so enormous that, even when using only 5 possible groups of processors and using tress with 30 operators, the LP file could not be opened in Cplex. For trees with 20 operators, Cplex produces the optimal solution, which consists in all cases in buying a single processor. So we opted for evaluating our heuristics vs. the optimal solution under homogeneous conditions, i.e., when there is a single processor type. In this case we skip the downgrade step after the server allocation. When α is less than 1, Subtree-bottom-up almost always finds the optimal solution (see Figure 9(a)). Note that once again, in two cases this heuristic is not able to find a feasible solution, while the others succeed ($N \in \{34, 35, 36\}$). This is again due to the fact that Subtree-bottom-up maps all operators onto a single processor and then the server association process fails. The other heuristics buy more processors from



(a) Each basic object available only on one server.

(b) Each basic object available on 50% of the servers.

Figure 8: Simulation to evaluate the influence of the availability of the basic objects.



(a) $\alpha = 0.9$.

(b) $\alpha = 1.1$.

Figure 9: Simulation to compare the heuristics' performances to the LP performance on homogeneous platforms.

the onset, and are later able to find a feasible processor-server association.

Even with homogeneous conditions, we observe the same ranking of our heuristics as before: Subtree-bottom up, the Greedy family, followed by Object-Grouping, then Object-Availability and finally Random. Focusing on the Greedy family, we observe that with increasing operator trees, Comp-Greedy outperforms Object-Greedy, and in most cases Comm-Greedy achieves the best costs of the three.

Summary Our results show that all our more sophisticated heuristics perform better than the simple random approach. Unfortunately, the object sensitive heuristics, Object-Grouping and Object-Availability, do not show the desired performance. We think that in some situations these heuristics could lead to good performance, but this is not observed on our set of random application configurations. We had found that Subtree-bottom-up outperforms other heuristics in most situations and also produces results very close to the optimal (for the cases in which we were able to determine the optimal). There are some configurations for which Subtree-bottom-up fails, our results suggest that on should use one of our Greedy heuristics, which perform reasonably well.

8 Conclusion

In this paper we have studied the problem of resource allocation for in-network stream processing. We formalized several operator-placement problems. We have focused more particularly on a "constructive" scenario in which one aims at minimizing the cost of a platform that satisfies an application throughput requirement. The complexity analysis showed that all problems are NP-complete, even for the simpler cases. We have derived an integer linear programming formulation of the various problems, and we have proposed several polynomial time heuristics for the constructive scenario. We compared these heuristics through simulation, allowing us to identify one heuristic that is almost always better than the others, Subtree-bottom-up. Finally, we assessed the absolute performance of our heuristics with respect to the optimal solution of the linear program for homogeneous platforms and small problem instances. It turns out that the Subtree-bottom-up heuristic almost always produces optimal results.

An interesting direction for future work is the study of the case when multiple applications must be executed simultaneously so that a given throughput must be achieved for each application. In this case a clear opportunity for higher performance with a reduced cost is the reuse of common sub-expression between trees [28, 29]. Another direction is the study of applications that are mutable, i.e., whose operators can be rearranged based on operator associativity and commutativity rules. Such situations arise for instance in relational database applications [10].

References

[1] B. Badcock, S. Babu, M. Datar, R. Motwani, and J. Widom, "Models and issues in data stream systems," in *Proceedings of the Intl. Conf. on Very Large Data Bases*, 2004, pp. 456–467.

[2] U. Srivastava, K. Munagala, and J. Widom, "Operator Placement for In-Network Stream Query Processing," in *Proceedings of the 24th Intl. Conf. on Principles of Database Systems*, 2005, pp. 250–258.

[3] S. Madden, M. Franklin, J. Hellerstein, and W. Hong, "The design of an acquisitional query processor for sensor networks," in *Proceedings of the 2003 ACM SIGMOD Intl. Conf. on Management of Data*, 2003, pp. 491–502.

[4] P. Bonnet, J. Gehrke, and P. Seshadri, "Towards sensor database systems," in *Proceedings of the Conference on Mobile Data Management*, 2001.

[5] C. Cranor, Y. Gao, T. Johnson, V. Shkapenyuk, and O. Spatscheck, "Gigascope: high-performance network monitoring with an SQL interface," in *Proceedings of the ACM SIGMOD International Conference on Management of Data*, 2002, pp. 623–633.

[6] R. van Rennesse, K. Birman, D. Dumitriu, and W. Vogels, "Scalable Management and Data Mining Using Astrolabe," in *Proceedings from the First International Workshop on Peer-to-Peer Systems*, 2002, pp. 280–294.

[7] E. Cooke, R. Mortier, A. Donnelly, P. Barham, and R. Isaacs, "Reclaiming Network-wide Visibility Using Ubiquitous End System Monitors," in *Proceedings of the USENIX Annual Technical Conference*, 2006.

[8] S. Babu and J. Widom, "Continuous Queries over Data Streams," *SIGMOD Record*, vol. 30, no. 3, 2001.

[9] L. Liu, C. Pu, and W. Tang, "Continual Queries for Internet Scale Event-Driven Information Delivery," *IEEE Transactions on Knowledge and Data Engineering*, vol. 11, no. 4, pp. 610–628, 1999.

[10] J. Chen, D. J. DeWitt, and J. F. Naughton, "Design and Evaluation of Alternative Selection Placement Strategies in Optimizing Continuous Queries," in *ICDE*, 2002. [Online]. Available: citeseer.ist.psu.edu/article/chen02design.html

[11] B. Plale and K. Schwan, "Dynamic Querying of Streaming Data with the dQUOB System," *IEEE Transactions on Parallel and Distributed Systems*, vol. 14, no. 4, pp. 422–432, 2003.

[12] J. Kräme and B. Seeger, "A Temporal Foundation for Continuous Queries over Data streams," in *Proceedings of the Intl. Conf. on Management of Data*, 2005, pp. 70–82.

[13] D. J. Abadi, Y. Ahmad, M. Balazinska, U. Cetintemel, M. Cherniack, J.-H. Hwang, W. Lindner, A. S. Maskey, A. Rasin, E. Ryvkina, N. Tatbul, Y. Xing, and S. Zdonik, "The Design of the Borealis Stream Processing Engine," in *Second Biennial Conference on Innovative Data Systems Research (CIDR 2005)*, Asilomar, CA, January 2005.

[14] M. Cherniack, H. Balakrishnan, M. Balazinska, D. Carney, U. Cetintemel, Y. Xing, and S. Zdonik, "Scalable distributed stream processing," in *Proc. of the CIDR Conf.*, January 2003.

[15] R. Huebsch, J. M. Hellerstein, N. L. Boon, T. Loo, S. Shenker, and I. Stoica, "Querying the Internet with PIER," Sept. 2003. [Online]. Available: citeseer.ist.psu.edu/huebsch03querying.html

[16] L. Chen, K. Reddy, and G. Agrawal, "GATES: a grid-based middleware for processing distributed data streams," *High performance Distributed Computing, 2004. Proceedings. 13th IEEE International Symposium on*, pp. 192–201, 4-6 June 2004.

[17] S. Nath, A. Deshpande, Y. Ke, P. B. Gibbons, B. Karp, and S. Seshan, "IrisNet: An Architecture for Internet-scale Sensing Services." [Online]. Available: citeseer.ist.psu.edu/nath03irisnet.html

[18] J. Chen, D. DeWitt, F. Tian, and Y. Wang, "NiagaraCQ: A scalable continuous query system for internet databases," in *Proceedings of the SIGMOD Intl. Conf. on Management of Data*, 2000, pp. 379–390.

[19] D. Logothetis and K. Yocum, "Wide-Scale Data Stream Management," in *Proceedings of the USENIX Annual Technical Conference*, 2008.

[20] P. Pietzuch, J. Leflie, J. Shneidman, M. Roussopoulos, M. Welsh, and M. Seltzer, "Network-Aware Operator Placement for Stream-Processing Systems," in *Proceedings of the 22nd International Conference on Data Engineering (ICDE'06)*, 2006, pp. 49–60.

[21] Y. Ahmad and U. Cetintemel, “Network aware query processing for stream-based applications,” in *Proceedings of the International Conference on Very Large Data Bases*, 2004, pp. 456–467.

[22] Y. E. Ioannidis, “Query optimization,” *ACM Computing Surveys*, vol. 28, no. 1, pp. 121–123, 1996. [Online]. Available: citeseer.ist.psu.edu/article/ioannidis96query.html

[23] S. Chaudhuri, “An Overview of Query Optimization in Relational Systems,” in *Proc. 17th ACM symposium on Principles of Database Systems*, 1998, pp. 34–43.

[24] A. Deshpande, Z. G. Ives, and V. Raman, “Adaptive query processing,” *Foundations and Trends in Databases*, vol. 1, no. 1, pp. 1–140, 2007.

[25] B. Hong and V. K. Prasanna, “Adaptive allocation of independent tasks to maximize throughput,” *IEEE Trans. Parallel Distributed Systems*, vol. 18, no. 10, pp. 1420–1435, 2007.

[26] M. R. Garey and D. S. Johnson, *Computers and Intractability, a Guide to the Theory of NP-Completeness*. W.H. Freeman and Company, 1979.

[27] “Source Code for the Heuristics.” [Online]. Available: http://graal.ens-lyon.fr/˜vsonigo/code/query-streaming/

[28] V. Pandit and H. Ji, “Efficient in-network evaluation of multiple queries,” in *HiPC*, 2006, pp. 205–216.

[29] K. Munagala, U. Srivastava, and J. Widom, “Optimization of continuous queries with shared expensive filters,” in *PODS ’07: Proceedings of the twenty-sixth ACM SIGMOD-SIGACT-SIGART symposium on Principles of database systems*. New York, NY, USA: ACM, 2007, pp. 215–224.