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Optimizing the translation out-of-SSA with renaming constraints

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August 2005
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Abstract

Static Single Assignment form is an intermediate representation that uses $\phi$ instructions to merge values at each confluent point of the control flow graph. $\phi$ instructions are not machine instructions and must be renamed back to move instructions when translating out of SSA form. Without a coalescing algorithm, the out of SSA translation generates many move instructions. Leung and George [8] use a SSA form for programs represented as native machine instructions, including the use of machine dedicated registers. For this purpose, they handle renaming constraints thanks to a pinning mechanism. Pinning $\phi$ arguments and their corresponding definition to a common resource is also a very attractive technique for coalescing variables. In this paper, extending this idea, we propose a method to reduce the $\phi$-related copies during the out of SSA translation, thanks to a pinning-based coalescing algorithm that is aware of renaming constraints. This report provides also a discussion about the formulation of this problem, its complexity and its motivations. We implemented our algorithm in the STMicroelectronics Linear Assembly Optimizer [5]. Our experiments show interesting results when comparing to the existing approaches of Leung and George [8], Sreedhar et al. [11], and Appel and George for register coalescing [7].

Keywords: Static Single Assignment, Coalescing, NP-complete, K-COLORABILITY, Machine code level, register allocation
Résumé
La forme SSA est une représentation intermédiaire de compilateur qui utilise des fonctions virtuelles $\phi$ pour fusionner les valeurs à chaque point de confluence du graphe de contrôle. Les fonctions $\phi$ n’existant pas physiquement, elles doivent être remplacées par des instructions `move` lors de la translation en code machine. Sans coalesceur, la translation hors-SSA génère beaucoup de `move`.
Dans cet article, nous proposons une extension de l’algorithme de Leung et George [8] qui effectue la minimisation de ces instructions de copie. Leung et al. proposent un algorithme de translation d’une forme SSA pour du code assembleur, mais non optimisé pour le remplacement des instructions $\phi$. Par contre, ils utilisent la notion d’épinglage pour représenter les contraintes de renommage.
Notre idée est d’utiliser cette notion d’épinglage afin de contraindre le renommage des arguments des $\phi$ pour faire du coalescing. C’est une formulation du problème de coalescing non équivalente au problème initial toujours considéré comme ouvert dans la littérature [8, 10]. Nous prouvons néanmoins la NP-complétude de notre formulation, une conséquence de la preuve étant la NP-complétude du problème initial en la taille de la plus grande fonction $\phi$.
Enfin, nous avons implémenté notre algorithme dans le LAO [5], optimiseur d’assembleur linéaire. La comparaison avec différentes approches possibles fournit de nombreux résultats intéressants. Nous avons aussi essayé, à l’aide d’examles faits à la main, d’expliquer les avantages et limitations des différentes approches.

**Mots-clés:** forme SSA, fusion de variables, NP-complétude, K-COLORABLE, code assembleur, allocation de registres
Optimizing the translation out-of-SSA with renaming constraints

1 Introduction

Static Single Assignment The Static Single Assignment (SSA) form is an intermediate representation widely used in modern compilers. SSA comes in many flavors, the one we use is the pruned SSA form \[4\]. In SSA form, each variable name, or virtual register, corresponds to a scalar value and each variable is defined only once in the program text. Because of this single assignment property, the SSA form contains $\phi$ instructions that are introduced to merge variables that come from different incoming edges at a confluent point of the control flow graph. These $\phi$ instructions have no direct corresponding hardware instructions, thus a translation out of SSA must be performed. This transformation replaces $\phi$ instructions with move instructions and some of the variables with dedicated ones when necessary. This replacement must be performed carefully whenever transformations such as copy propagation have been done while in SSA form. Moreover, a naive approach for the out of SSA translation generates a large number of move instructions. This paper \[1\] addresses the problem of minimizing the number of generated copies during this translation phase.

Previous Work Cytron et al. \[4\] proposed a simple algorithm that first replaces a $\phi$ instruction by copies into the predecessor blocks, then relies on Chaitin’s coalescing algorithm \[3\] to reduce the number of copies. Briggs et al. \[1\] found two correctness problems in this algorithm, namely the swap problem and the lost copy problem, and proposed solutions to these. Sreedhar et al. \[11\] proposed an algorithm that avoids the need for Chaitin’s coalescing algorithm and that can eliminate more move instructions than the previous algorithms. Leung and George \[8\] proposed an out-of-SSA algorithm for an SSA representation at the machine code level. Machine code level representations add renaming constraints due to ABI (Application Binary Interface) rules on calls, special purpose ABI defined registers, and restrictions imposed on register operands.

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\[1\]Many thanks to Alain Darte, Stephen Clarke, Daniel Grund and the reviewers of CGO for very helpful comments on the presentation of this report.
Context of the study  Our study of out-of-SSA algorithms was needed for the development of the STMicroelectronics Linear Assembly Optimizer (LAO) tool. LAO converts a program written in the Linear Assembly Input (LAI) language into the final assembly language that is suitable for assembly, linking, and execution. The LAI language is a superset of the target assembly language. It allows symbolic register names to be freely used. It includes a number of transformations such as induction variable optimization, redundancy elimination, and optimizations based on range propagation, in an SSA intermediate representation. It includes scheduling techniques based on software pipelining and superblock scheduling, and uses a repeated coalescing [5] register allocator, which is an improvement over the iterated register coalescing from George and Appel [7]. The LAO tool targets the ST120 processor, a DSP processor with full predication, 16-bit packed arithmetic instructions, multiply-accumulate instructions and a few 2-operands instructions such as addressing mode with auto-modification of base pointer.

Because of these particular features, an out-of-SSA algorithm aware of renaming constraints was needed. In fact, delaying renaming constraints after the out-of-SSA phase would result in additional move instructions (see Section 5), along with possible infeasibilities and complications. We modified an out-of-SSA algorithm from Leung and George to handle renaming constraints and reduce the number of move instructions due to the replacement of \( \phi \) instructions.

Layout of this paper  The paper is organized as follows. Section 2 states our problem and gives a brief description of Leung and George’s algorithm. In Section 3, we present our solution to the problem of register coalescing during the out-of-SSA phase. Section 4 discusses, through several examples, how our algorithm compares to others. In Section 5, we present results that show the effectiveness of our solution on a set of benchmarks, and we finally conclude. This paper contains also two appendices A and B devoted respectively to the refinement of Leung’s algorithm and to the NP-completeness proof of the pinning based coalescing problem.

2 Problem statement and Leung and George’s algorithm

Our goal is to handle renaming constraints and coalescing opportunities during the out of SSA translation. For that, we distinguish dedicated registers (such as R0 or SP, the stack pointer) from general-purpose registers that we assume in an unlimited number (we call them virtual registers or variables). We use a pinning mechanism, in much the same way as in Leung and George’s algorithm [8], so as to enforce the use of these dedicated registers and to favor coalescing. Then, constraints on the number of general-purpose registers are handled later, in the register allocation phase.

2.1 Pinning mechanism

An operand is the textual use of a variable, either as a write (definition of the variable) or as a read (use in an instruction). A resource is either a physical register or a variable. Resource pinning or simply pinning is a pre-coloring of operands to resources. We call variable pinning the pinning of the (unique) definition of a variable. Due to the semantics of \( \phi \) instructions, all arguments (i.e. use operands) of a \( \phi \) instruction are pinned to the same resource as the variable defined (i.e. def operand) by the \( \phi \).

On the ST120 processor, as in Leung and George’s algorithm, we have to handle Instruction Set Architecture (ISA) register renaming constraints and Application Binary Interface (ABI) function parameter passing rules. Figure 2, expressed in SSA pseudo assembly code, gives an example of such constraints. In this example and in the rest of this paper, the notation \( X \stackrel{R}{\uparrow} \) is used to mark that the
operand X is pinned to the resource R. When the use of a variable is pinned to a different resource than its definition, a move instruction has to be inserted between the resource of the definition and the resource of the use. Pinning the variable to the same resource as its uses has the effect of coalescing these resources (i.e., it deletes the move).
motivation for these semantics is given in Appendix A. Finally, Case 6 corresponds to a more subtle incorrect pinning, similar to the problem stressed in Figure 3.

\[
\begin{align*}
\text{R}_{0} = x_{3} &+ K \\
x_{4} &+ y_{1} + K \\
\text{return } x_{3} &+ R_{0}
\end{align*}
\]

\[
\begin{align*}
\text{R}_{0} &+ R_{1} \\
R_{0} &+ R_{1} \\
\text{return } R_{0}
\end{align*}
\]

Figure 4: Transformation of already pinned SSA code by Leung and George’s algorithm.

Case 1: \((x\uparrow r, y\uparrow r) = \text{instr(...)})

Case 2: \(\ldots \text{instr}(x\uparrow r, y\uparrow r))

Case 3: \(x\uparrow r = \phi(...) \quad y\uparrow r = \phi(...)\)

Case 4: \(x\uparrow r = \text{instr}(y\uparrow r)\)

Case 5: \(x\uparrow r = \phi(...) \quad y\uparrow r = \phi(...)\)

Case 6: \(x\uparrow r = \phi(R_{0}, R_{1}) \quad y\uparrow r = \phi(...) \quad x\uparrow r = (x_{1}, \ldots)\)

Figure 5: All but Case 4 are incorrect pinning.

2.3 Leung and George’s algorithm

Leung and George’s algorithm is decomposed into three consecutive phases: the collect phase collects information about renaming constraints; the mark phase collects information about the conflicts generated by renaming; the reconstruct phase performs renaming, inserts copies when necessary and replaces \(\phi\) instructions.

Pinning occurs during the collect phase, and then the out of SSA translation relies on the mark and reconstruct phases. Figure 4 illustrates the transformations performed during those last two phases:

- \(x_{3}\) is pinned to \(R_{0}\) on its definition. But, on the path to its use in the \text{return}, \(x_{4}\) is also pinned to \(R_{0}\) on the call to \(g\). We say that \(x_{3}\) is killed, and a repair copy to a new variable \(x'_{3}\) is introduced.

- The use of \(x_{3}\) in the call to \(g\) is pinned to \(R_{0}\), while \(x_{3}\) is already available in \(R_{0}\) due to a prior pinning on the \(\phi\) instruction. The algorithm is careful not to introduce a redundant move instruction in this case.

- The copies \(R_{0} = R_{1}; R_{1} = R_{0}\) are performed in parallel in the algorithm, so as to avoid the so-called swap problem. To sequentialize the code, intermediate variables may be used and the copies may be reordered, resulting in the code \(t = R_{1}; R_{1} = R_{0}; R_{0} = t\) in this example.
Now, consider the non-pinned variable \( y_2 \) of Figure 4 and its use in the definition of \( x_4 \). The use is pinned to a resource, \( R_1 \), and \( y_2 \) could have been coalesced to \( R_1 \) without creating any interference. The main limitation of Leung and George’s algorithm is its inability to do so. The same weakness shows up on \( \phi \) arguments, as illustrated by Figure 6(a): on a \( \phi \) instruction \( X = \phi(x_0, \ldots, x_n) \), each operand \( x_i \) is implicitly pinned to \( X \), while the definition of each \( x_i \) may not. Our pinning-based coalescing is an extension to the pinning mechanism whose goal is to overcome this limitation.

### 2.4 The \( \phi \) coalescing problem

As opposed to the pinning due to ABI constraints, which is applied to a textual use of an SSA variable, the pinning related to coalescing is applied only to variable definitions (variable pinning). Figure 6 illustrates how this pinning mechanism can play the role of a coalescing phase by preventing the reconstruction phase of Leung and George’s algorithm from inserting move instructions: in Figure 6(b), \( x_1 \) and \( x_2 \) were pinned to \( x \) to eliminate these move instructions; however, this pinning creates an interference, which results in a repair move \( x' = x \) along with a move \( x = x' \) on the replacement of the \( \phi \) instruction; in Figure 6(c), to avoid the interference, only \( x_2 \) was pinned to \( x \), resulting in only one move instruction.

![Figure 6: Inability of Leung and George’s algorithm to coalesce](image)

<table>
<thead>
<tr>
<th>Initial SSA form</th>
<th>(a) Final code if nothing pinned</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x = \phi(x_1, x_2) )</td>
<td>( x_1 = \text{exp}_1 )</td>
</tr>
<tr>
<td>( x = \text{exp}_1 )</td>
<td>( x = \text{exp}_2 )</td>
</tr>
<tr>
<td>save ( x \rightarrow x' = x )</td>
<td>( x_1 = \text{exp}_1 )</td>
</tr>
<tr>
<td>conflict ( \ldots )</td>
<td>( x_2 \text{ pinned to } x )</td>
</tr>
<tr>
<td>restore ( x \leftarrow x' )</td>
<td>( x = x_1 )</td>
</tr>
<tr>
<td>(b) Final code if ( x, x_1 ) ( x_2 ) are pinned to the virtual resource ( x )</td>
<td>no copy needed</td>
</tr>
<tr>
<td>(c) Final code if ( x ) and ( x_2 ) are pinned to the virtual resource ( x )</td>
<td></td>
</tr>
</tbody>
</table>

Therefore, we will only look for a variable pinning that does not introduce any new interference. In this case, for a \( \phi \) instruction \( X = \phi(x_0, \ldots, x_n) \), we say that the gain for \( \phi \) is the number of indices \( i \) such that the variable \( x_i \) is pinned to the same resource as \( X \). Hence, our \( \phi \) coalescing problem consists of finding a variable pinning, with no new interference (i.e., without changing the number of variables for which a repair move is needed), that maximizes the total gain, taking into account all \( \phi \) instructions in the program.

#### Algorithm 1 Main phases of our algorithm.

```plaintext
Program_pinning(CFG_Program P)
foreach basic block B in P, in an inner to outer loop traversal
   Initial_G=Create_affinity_graph(B)
   PrePruned_G=Graph_InitialPruning(Initial_G)
   Final_G=BipartiteGraph_pinning(PrePruned_G)
   PrunedGraph_pinning(Final_G)
```


3 Our solution

The $\phi$ coalescing problem we just formulated is NP-complete (see Appendix B for details). Instead of trying to minimize the gain for all $\phi$ instructions together, our solution relies on a sequence of local optimizations, each one limited to the gain for all $\phi$ instructions defined at a confluence point of the program. These confluence points are traversed based on an inner to outer loop traversal, so as to optimize in priority the most frequently executed blocks. The skeleton of our approach is given in Algorithm 1.

Let us first describe the general ideas of our solution, before entering the details. For an SSA variable $y$, we define $y = \text{Resource}_{\text{def}}(y)$ as $r$ if the definition of $y$ is pinned to $r$, or $y$ otherwise. Also, for simplicity, we identify the notion of resource with the set of variables pinned to it. For a given basic block, we create what we call an affinity graph. Vertices represent resources; edges represent potential copies between variables that can be coalesced if pinned to the same resource. Edges are weighted to take into account interferences between SSA variables; then the graph is pruned (deleting in priority edges with large weights) until, in each resulting connected component, none of the vertices interfere: they can now be all pinned to the same resource. The rest of this section is devoted to the precise description of our algorithm. A pseudo code is given in Algorithm 2 on page 17. The consecutive steps of this algorithm are applied on the example of Figure 8.

3.1 The initial affinity graph

For a given basic block, the affinity graph is an undirected graph where each vertex represents either a variable or its corresponding resource (if already pinned): two variables that are pinned to the same resource are collapsed into the same vertex. Then, for each $\phi$ instruction $X = \phi(x_1, \ldots, x_n)$ at the entry of the basic block, there is an affinity edge, for each $i$, $0 \leq i \leq n$, between the vertex that contains $X$ and the vertex that contains $x_i$.

3.2 Interferences between variables

We define below four classes of interferences that can occur when pinning two operands of a $\phi$ instruction to the same resource. We differentiate simple interferences from strong interferences: a strong interference generates an incorrect pinning. On the other hand, a simple interference can always be repaired despite the fact that the repair might generate additional copies. The goal is then to minimize the number of simple interferences and to avoid all strong interferences. The reader may find useful to refer, for each class, to Figure 7.

[Class 1] Consider two variables $x$ and $y$. If there exists a point in the control flow graph where both $x$ and $y$ are alive, then $x$ and $y$ interfere. Moreover, considering the definitions of $x$ and $y$, one dominates the other (this is a property of the SSA form). If the definition of $x$ dominates the definition of $y$, we say that the definition of $x$ is killed by $y$. The consequence is that pinning the definitions of $x$ and $y$ to a common resource would result in a repair of $x$ (as in Leung and George’s technique).

[Class 2] Consider a $\phi$ instruction $y = \phi(\ldots, z, \ldots)$ in basic block $B$. Let $C$ be the block where the argument $z$ comes from; textually, the use of $z$ appears in block $B$ (and is implicitly pinned to $y$), but semantically, it takes place at the end of basic block $C$ (this is where a move instruction, if needed, would be placed). If $x \neq z$ and $x$ is live-out of block $C$, then $x$ and the use of $z$ interfere and we say that the definition of $x$ is killed by $y$. Note our definition of liveness: a $\phi$ instruction does not occur where it textually appears, but at the end of each predecessor basic block instead. Hence, if not
used by another instruction, \( z \) would be treated as dead at the exit of block \( C \) and at the entry of block \( B \).

[Class 3] Consider two variables \( x \) and \( y \), both defined by \( \phi \) instructions, but not necessarily in the same basic block. Some of their respective arguments (for example \( x_i \) and \( y_j \)) may interfere in a common predecessor block \( B \). In this case, we say that the definitions of \( x \) and \( y \) **strongly interfere**: indeed, as explained in Section 2.2, pinning those two definitions together is incorrect.

[Class 4] Consider \( \phi \) instructions \( y = \phi(y_1, \ldots, y_n) \) and \( z = \phi(z_1, \ldots, z_n) \), in the same basic block and with the same arguments. Because of Leung and George’s repairing implementation, they cannot be considered as identical and we need to consider that they strongly interfere. Notice that a redundancy elimination algorithm should have eliminated this case before. Note that, by definition of Classes 3 and 4, all variables defined by \( \phi \) instructions in the same basic block strongly interfere.

Also, we consider that variables pinned to two different physical registers strongly interfere.

\[
\begin{align*}
\text{Class 1:} & \quad x = \ldots \quad y = \ldots \\
\text{Class 2:} & \quad x_2 \neq y_1 \\
\text{Class 3 & 4:} & \quad x \text{ strongly interferes with } y \\
& \quad y \text{ strongly interferes with } z
\end{align*}
\]

Figure 7: Different kind of interferences between variables.

Figure 8: Program pinning on an example. Control-flow graphs are represented for code, with control-flow edges between basic blocks represented with solid black arrows. Affinity graphs are represented for step 1 and 2, with affinity edges represented as dashed gray lines, annotated with a weight, and with interferences edges represented as full gray lines, annotated with the class of the interference.
3.3 Interferences between resources

After the initial pinning (taking into account renaming constraints), a resource cannot contain two variables that strongly interfere. However, simple interferences are possible; they will be solved by Leung and George’s repairing technique. During our iterative pinning process, we keep merging more and more resources, but we make sure not to create any new interference. We say that two resources \( A = \{x_1, \ldots, x_n\} \) and \( B = \{y_1, \ldots, y_m\} \) interfere if pinning all the variables \( \{x_1, \ldots, x_n\} \) and \( \{y_1, \ldots, y_m\} \) together creates either a new simple interference, or a strong interference, i.e., if there exist \( x_i \) and \( y_j \) that interfere. This check is done by the procedure \texttt{Resource\_interfere}; it uses the procedure \texttt{Resource\_killed} that gives, within a given resource, all the variables already killed by another variable. \texttt{Resource\_killed} is given in a formal description, but obviously the information can be maintained and updated after each merge.

3.4 Pruning the affinity graph

The pruning phase is based on the interference analysis between resources. More formally, the optimization problem can be stated as follows. Let \( G = (V, E_{\text{Affinity}}) \) be the graph obtained from \texttt{Create\_affinity\_graph} (as explained in Section 3.1): the set \( V \) is the set of vertices labeled by resources and \( E_{\text{Affinity}} \) is the set of affinity edges between vertices. The goal is to prune (edge deletion) the graph \( G \) into \( G' = (V, E_{\text{pinned}}) \) such that:

\textbf{Condition 1:} the cardinality of \( E_{\text{pinned}} \) is maximized;

\textbf{Condition 2:} for each pair of resources \((v_1, v_2) \in V^2\) in the same connected component of \( G' \), \( v_1 \) and \( v_2 \) do not interfere, i.e., \texttt{Resource\_interfere}(\( v_1, v_2 \)) = false.

In other words, the graph \( G \) is pruned into connected components such that the total number of deleted edges from \( E_{\text{Affinity}} \) is minimized and no two resources within the same connected component interfere.

First, because of \textbf{Condition 2}, all edges \((v_1, v_2) \in E_{\text{Affinity}}\) such that \( v_1 \) and \( v_2 \) interfere need to be removed from \( G \). The obtained graph \texttt{PrePruned\_G} is bipartite. Indeed, let \( \{X_i\}_{1 \leq i \leq m} \), with \( X_i = \phi(x_{i,1}, \ldots, x_{i,n}) \), be the set of \( \phi \) instructions of the current basic block \( B \). There are two kinds of vertices in \( G \), the vertices for the definitions \( V_{\text{DEFS}} = \{\texttt{Resource\_def}(X_i)\}_{1 \leq i \leq m} \) and the other ones, for the arguments not already in \( V_{\text{DEFS}} \), \( V_{\text{ARGS}} = \{\texttt{Resource\_def}(x_{i,j})\}_{1 \leq i \leq m, 1 \leq j \leq n} \setminus V_{\text{DEFS}} \). By construction, there is no affinity edge between two elements of \( V_{\text{ARGS}} \). Also, because elements of \( V_{\text{DEFS}} \) strongly interfere together, there remains no edge between two elements of \( V_{\text{DEFS}} \). Thus, \( G \) is indeed bipartite.

Unfortunately, even for a bipartite affinity graph, the pruning phase is NP-complete in the number of \( \phi \) instructions (see Appendix B). Therefore, we use a heuristic algorithm based on a greedy pruning of edges, where edges with large weights are chosen first. The weight of an edge \((x, y)\) is the number of neighbors of \( x \) (resp. \( y \)) that interfere with \( y \) (resp. \( x \)). This has the effect of first deleting edges that are more likely to disconnect more interfering vertices (see details in the procedure \texttt{Bipartite-Graph\_pruning}). Note that, in the particular case of a unique \( \phi \) instruction, this is identical to the “Process the unresolved resources” of the algorithm of Sreedhar et al. [11].

3.5 Merging the connected components

Once the affinity graph has been pruned, the resources of each connected component can be merged. We choose a reference resource in this connected component, either the physical resource if it exists (in this case, it is unique since two physical resources always interfere), or any resource otherwise. We change all pinnings to a resource of this component into a pinning to the reference resource. Finally,
we pin each variable (i.e., its definition) in the component to this reference resource. The correctness of this phase is insured by the absence of any strong interference inside the new merged resource. A formal description of the algorithm is given by the procedure `PrunedGraph.pinning`. In practice, the update of pinning need be performed only once, just before the mark phase, so requiring only one traversal of the control flow graph. Also note that the interference graph can be built incrementally at each call to `Resource.interfere` and updated at each resource merge, using a simple vertex-merge operation: hence, as opposed to the merge operation used in the iterated register coalescing algorithm [7] where interferences have to be recomputed at each iteration, here each vertex represents a SSA variable and merging is a simple edge union.

We point out that, after this phase, our algorithm relies on the mark and reconstruct phases of Leung and George’s algorithm. But we use several refinements, whose details are given in Appendix A.

4 Theoretical discussion

We now compare our algorithm with previous approaches, through hand crafted examples.

4.1 Our algorithm versus register coalescing

The out-of-SSA algorithm of Briggs et al. [1] relies on a Chaitin-style register coalescing to remove `move` instructions produced by the out of SSA translation. ABI constraints for a machine code level intermediate representation can be handled after the out of SSA translation by insertion of `move` instructions at procedure entry and exit, around function calls, and before 2-operand instructions. However, several reasons favor combined processing of coalescing and ABI renaming during the out-of-SSA phase:

[CC1] SSA is a higher level representation that allows a more accurate definition of interferences. For example (see Figure 9), it allows partial coalescing, i.e., the coalescing of a subset of the variable definitions.

Figure 9: Because the physical register $R0$ and $z$ interfere, [Initial code] cannot be coalesced by Chaitin’s register coalescing; even if the three definitions of $R0$ are constrained to be done on $R0$ (and then even in SSA “$R0$” and “$z$” interfere), the pinning mechanism allows $z$ and $R0$ to be coalesced, we say partially.

[CC2] The classical coalescing algorithm is greedy, so it may block further coalescings. Instead, for each merging point of the control flow graph, our algorithm optimizes together the set of coalescing opportunities for the set of $\phi$ instructions of this point.

[CC3] The main motivation of Leung and George’s algorithm is that ABI constraints introduce many additional `move` instructions. Some of these will be deleted by a dead code algorithm, but most of them will have to be coalesced. An important point of our method is the reduction of the overall complexity of the out-of-SSA renaming and coalescing phases: as explained in Section 3.5,
the complexity of the coalescings performed under the SSA representation benefits from the static single definition property.

### 4.2 Our algorithm versus the algorithm of Sreedhar et al.

The technique of Sreedhar et al. [11] consists in first translating the SSA form into CSSA (Conventional SSA) form. In CSSA, it is correct to replace all variable names that are part of a common $\phi$ instruction by a common name, then to remove all $\phi$ instructions. To go from SSA to CSSA however, we may create new variables and insert `move` instructions to eliminate $\phi$ variable interferences that would otherwise result in an incorrect program after renaming. Sreedhar et al. propose three algorithms to convert to CSSA form. We only consider the third one, which uses the interference graph and some liveness information to minimize the number of generated `move` instructions. Figures 10-12 illustrate some differences between the technique of Sreedhar et al. and ours.

[CS1] Sreedhar et al. optimize separately the replacement of each $\phi$ instruction. Our algorithm considers all the $\phi$ instructions of a given block to be optimized together. This can lead to a better solution as shown in Figure 10.

![Figure 10](image)

Figure 10: Sreedhar et al. treat $S_1$ and $S_2$ in sequence: for $S_1$, $\{x, y\}$ interfere so $X = x$ is inserted and $\{y, X\}$ are regrouped in the resource $X$; for $S_2$, $\{z, X\}$ interfere so $Y = X$ is inserted and $Y = \{z, Y\}$.

![Figure 11](image)

Figure 11: The superiority of using parallel copies. For the solution of Sreedhar et al. we suppose $S_1, S_2, S_3$ and $S_4$ were treated in this order.
[CS2] Sreedhar et al. process *iteratively* modify the initial SSA code by splitting variables. By doing so, information on interferences becomes scattered and harder to use. Thanks to pinning, throughout the process we are always reasoning on the initial SSA code. In particular, as illustrated by Figure 11, we can take advantage of the parallel copies placement.

[CS3] Finally, because our SSA representation is at machine level, we need to take into account ABI constraints. Figure 12 shows an example where we make a better choice of which variables to coalesce by taking the ABI constraints into account.

\[
\begin{align*}
  b_0 &= f_1 \\
  b_1 &= \phi(b_0, B) \\
  b_2 &= b_1 + 1 \ (\text{autoadd}) \\
  a &= \ldots \\
  L_1: & \quad L_2: \\
  B &= \phi(a, b_2) \\
  B &= f_1 \\
  B &= f_1 \\
  \{b_1 \text{ is in } B\} \\
  b_2 &= B_1 + 1 \ (\text{autoadd}) \\
  B &= \ldots \\
  \{a \text{ is in } B\} \\
  b_2 &= B \\
  B+ &= 1 \\
  a &= \ldots \\
  \{B, b_2\} \text{ interfere} \\
  B &= b_2 \\
  B &= a
\end{align*}
\]

Figure 12: \(\{a, b_2\}\) interfere: without the ABI constraints information, adding the move on block \(L_1\) or \(L_2\) is equivalent. Sreedhar et al. may make the wrong choice: treating the ABI afterward would replace the autoadd into \(B = B + 1; b_2 = B\) (because \(\{B, b_2\}\) interfere) resulting in one more move.

### 4.3 Limitations

Below are several points that expose the limitations of our approach:

[LIM1] Our algorithm is based on Leung and George’s algorithm that decides the place where move instructions are inserted. Also, we use an approximation of the cost of an interference compared to the gain of a pinning. Hence, even if we could provide an optimal solution to our formulation of the problem, this solution would not necessarily be an optimal solution for the minimization of move instructions.

[LIM2] As explained in Section 2.3, the main limitation of Leung and George’s algorithm is that, when the use of a variable is pinned to a resource, it does not try to coalesce its definition with this resource. This can be avoided by using a pre-pass to pin the variable definitions. But, as illustrated by Figure 13, repairing variables that are introduced during Leung and George’s repairing phase cannot be handled this way.

[LIM3] As explained in Appendix B, our \(\phi\) coalescing problem is NP-complete. Note also that a simple extension of the proof shows the NP-completeness of the problem of minimizing the number of move instructions.

[LIM4] Finally, in the case of strong register pressure, the problem becomes different: coalescing (or splitting) variables has a strong impact on the colorability of the interference graph during the register allocator phase (e.g. [9]). But this goes out of the scope of this paper.
$R_0 = x \quad \rightarrow \quad \phi(x_0, x_1)$

$x_1 = x + 1$

$\cdots = f(x_0 \rightarrow R_0)$

$[\text{Initial SSA code}]$

$x = \ldots$

$x' = x \text{(repair)}$

$x = x + 1$

$R_0 = x \text{(repair)}$

$\cdots = f(R_0)$

$[\text{Our solution}]$

$x = \ldots$

$x_1 = x + 1$

$\cdots = f(x \uparrow R_0)$

$[\text{Optimal solution}]$

Figure 13: Limitation of Leung and George’s repairing process: the repairing variable $x'$ is not coalesced with further uses.

5 Results

We conducted our experiments on several benchmarks represented in LAI code. Since the LAI language supports predicated instructions, the LAO tool uses a special form of SSA, named $\psi$-SSA [13], which introduces $\psi$ instructions to represent predicated code under SSA. In brief, $\psi$ instructions introduce constraints similar to 2-operands constraints, and are handled in our algorithm in a special pass where they are converted into a “$\psi$-conventional” SSA form. This does not change the results presented in this section.

In the following, VALcc1 and VALcc2 refer to the same set of C functions compiled into LAI code with two different ST120 C compilers. This set includes about 40 small functions with some basic digital signal processing kernels, integer Discrete Cosine Transform, sorting, searching, and string searching algorithms. The benchmarks example1-8 are small examples written in LAI code specifically for the experiment. LAI_Large is a set of larger functions, most of which come from the efr 5.1.0 vocoder from the ETSI [6]. Finally, SPECint refers to the SPEC CINT2000 benchmark [12].

To show the superiority of our approach, we have implemented the following algorithms:

[Leung] The algorithm of Leung and George contains the collect, and the mark-reconstruct (say out-of-pinned-SSA) phases. For some reasons given further, the collect phase has been split into three parts, namely pinning$_{SP}$ (collect constraints related to the dedicated register $SP$), pinning$_{ABI}$ (collect remaining renaming constraints) and pinning$_{\phi}$ (our algorithm). Each of these pinning phases can be activated or not, independently.

[Sreedhar] The algorithm of Sreedhar et al. has been implemented with an additional pass, namely pinning$_{CSSA}$. The pinning$_{CSSA}$ phase pins all the operands of a $\phi$ to a same resource, and allows the out-of-pinned-SSA phase to be used as an out-of-CSSA algorithm.

[Naive$\mathbf{ABI}$] is an algorithm that adds when necessary move instructions locally around renaming constrained instructions. This pass can be used when the pinning$_{ABI}$ pass has not been activated.

[Coalescing] Finally, we have implemented a repeated register coalescer [5]. As for the iterated register coalescer it is a conservative coalescer when used during the register allocation phase. But, outside of the register allocation context like here, it is an aggressive coalescing that does not take care of the colorability of the interference graph.

As already mentioned in Section 2.2, coalescing variables constrained by a dedicated register like the $SP$ register can generate incorrect code. Similarly, splitting the SSA web of such variables poses some problems. Hence, it was not possible to ignore those renaming constraints during the out-of-SSA phase and to treat them afterwards. That explains the differentiation we made between pinning$_{SP}$ and pinning$_{ABI}$ passes: we choose to always execute pinning$_{SP}$. Also, we tried to modify the algorithm of Sreedhar et al. to support $SP$ register constraints. However, our implementation still performs some illegal variable splitting on some codes: the final non-SSA code contains fewer move
instructions, but is incorrect. Such cases mainly occurred with SPECint, and thus the SPECint figures for the experiments including the algorithm of Sreedhar et al. must be taken only as an optimistic approximation of the number of move instructions.

Table 1: Details of implemented versions

<table>
<thead>
<tr>
<th>Experiments</th>
<th>Sreedhar</th>
<th>pinningCSSA</th>
<th>pinning_css</th>
<th>pinning_ali</th>
<th>pinning_ali</th>
<th>out-of-pinned-SSA</th>
<th>Naive</th>
<th>Coalescing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{\phi} + C$</td>
<td>• • • •</td>
<td>• • • •</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
<tr>
<td>$S_{\phi} + C$</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
<td>•</td>
</tr>
</tbody>
</table>

Table 2: Comparison of move instruction count with no ABI constraint.

Comparison without ABI constraints Table 2 compares different approaches when renaming constraints are ignored. As explained above only non-SP register related constraints, which we improperly call ABI constraints, could be ignored in practice. Columns $S_{\phi} + C$ vs $L_{\phi} + C$ illustrate points [CS1-2]. Columns C vs $L_{\phi} + C$ illustrate points [CC1-2]. Point [CC1] is also illustrated by $S_{\phi} + C$ vs C. In those experiments, our algorithm is better or equal in all cases, except for the SPECint benchmark with the algorithm of Sreedhar et al.. But $S_{\phi} + C$ are optimistic results as explained before. It shows the superiority of our approach in the absence of ABI constraints.

<table>
<thead>
<tr>
<th>benchmark</th>
<th>$L_{\phi} + C$</th>
<th>C</th>
<th>$S_{\phi} + C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VALcc1</td>
<td>193</td>
<td>+59</td>
<td>+3</td>
</tr>
<tr>
<td>VALcc2</td>
<td>170</td>
<td>+44</td>
<td>+13</td>
</tr>
<tr>
<td>example1-8</td>
<td>14</td>
<td>+3</td>
<td>+3</td>
</tr>
<tr>
<td>LAlarge</td>
<td>438</td>
<td>+44</td>
<td>+48</td>
</tr>
<tr>
<td>SPECint</td>
<td>6803</td>
<td>+3135</td>
<td>-59</td>
</tr>
</tbody>
</table>

Table 3 shows the variation in the number of move instructions of various out-of-SSA register coalescing algorithms, when all renaming constraints are taken into account. Comparison of $S_{\phi} + LAB1 + C$ and $LAB1 + C$ vs $L_{\phi, ABI} + C$ confirms points [CS3]. Column C shows the importance of treating the ABI with the algorithm of Leung et al.: many move instructions could not be removed by the dead code and aggressive coalescing phases. Our algorithm leads to less move instructions in all cases which shows the superiority of our approach with renaming constraints.
Compilation time  Repeated register coalescing is an expensive optimization phase in terms of time and space; its complexity is proportional to the number of move instructions in the program. Almost all coalescings are handled by our algorithm during the out of SSA translation. As explained in [2] the creation and the maintenance of the interference graph is highly simplified under the SSA form. Hence, as mentioned in Point [CC3], the more move instructions are handled at the SSA level, the lower is the compilation time for the overall coalescing. Table 4 gives an evaluation of the number of move instructions that would remain after the out-of-SSA phase if only naive techniques were applied for the $\phi$ replacement (which we denote $\phi$ moves) and for the renaming constraints treatment (which we improperly call ABI moves). Hence, it gives an evaluation of the cost of running a repeated register coalescing after one simple SSA rename back phase. We did not provide timing figures for the overall out-of-SSA and register coalescing phase for the different experiments because our implementation is too experimental and not optimized enough to give usable results.

Variations on our algorithm  Table 5 compares small variations in the implementation of our algorithm. The base column reports weighted move count, where move instructions are given a weight equal to $5^d$, $d$ being the nesting level, i.e. depth, of the loop the move belongs to. $5^d$ is an arbitrary

### Table 3: Comparison of move instruction count with renaming constraints.

<table>
<thead>
<tr>
<th>benchmark</th>
<th>$L_{\phi,ABI}$</th>
<th>$S_{\phi}$</th>
<th>$L_{ABI}$</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>VALcc1</td>
<td>242</td>
<td>+7</td>
<td>+3</td>
<td>+386</td>
</tr>
<tr>
<td>VALcc2</td>
<td>220</td>
<td>+15</td>
<td>+29</td>
<td>+449</td>
</tr>
<tr>
<td>example1-8</td>
<td>15</td>
<td>+3</td>
<td>+3</td>
<td>+18</td>
</tr>
<tr>
<td>LAI_Large</td>
<td>1085</td>
<td>+26</td>
<td>+62</td>
<td>+634</td>
</tr>
<tr>
<td>SPECint</td>
<td>23930</td>
<td>+413</td>
<td>+482</td>
<td>+38623</td>
</tr>
</tbody>
</table>

### Table 4: Comparison of move instruction count before the repeated register coalescing phase.

<table>
<thead>
<tr>
<th>benchmark</th>
<th>$L_{\phi,ABI}$</th>
<th>$S_{\phi}$</th>
<th>$L_{ABI}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>VALcc1</td>
<td>277</td>
<td>+593</td>
<td>+690</td>
</tr>
<tr>
<td>VALcc2</td>
<td>245</td>
<td>+926</td>
<td>+749</td>
</tr>
<tr>
<td>example1-8</td>
<td>16</td>
<td>+38</td>
<td>+34</td>
</tr>
<tr>
<td>LAI_Large</td>
<td>1447</td>
<td>+4543</td>
<td>+6161</td>
</tr>
<tr>
<td>SPECint</td>
<td>36882</td>
<td>+249481</td>
<td>+260095</td>
</tr>
</tbody>
</table>

### Table 5: Weighted count of move instructions on variants of our algorithm.

<table>
<thead>
<tr>
<th>benchmark</th>
<th>base</th>
<th>depth</th>
<th>opt</th>
<th>pess</th>
</tr>
</thead>
<tbody>
<tr>
<td>VALcc1</td>
<td>1109</td>
<td>+1</td>
<td>+4</td>
<td>+1484</td>
</tr>
<tr>
<td>VALcc2</td>
<td>877</td>
<td>+1</td>
<td>+8</td>
<td>+1716</td>
</tr>
<tr>
<td>example1-8</td>
<td>32</td>
<td>+0</td>
<td>+0</td>
<td>+4</td>
</tr>
<tr>
<td>LAI_Large</td>
<td>17594</td>
<td>+60</td>
<td>+7</td>
<td>+22116</td>
</tr>
<tr>
<td>SPECint</td>
<td>1652065</td>
<td>-1798</td>
<td>+7258</td>
<td>+3038712</td>
</tr>
</tbody>
</table>
weight that corresponds to a static approximation where each loop would contain 5 iterations.

Our first variation \( \text{(depth)} \) is based on the simple remark that in our initial implementation we prioritized the \( \phi \) instructions according to their depth, instead of the depth of the \text{move} instructions they will generate. For this variation, we use a new \text{Create affinity graph} procedure (Algorithm 3) with a depth constraint that calls \text{Program pinning} with decreasing depth. This leads to a very small improvement on SPECint and a small degradation for LAI_Large. This result confirms the observation we made that affinity and interference graphs are not complex enough to motivate a global optimization scheme.

Our second \( \text{(opt)} \) and third \( \text{(pess)} \) variations use relaxed definitions of interferences, respectively \text{optimistic} and \text{pessimistic} (Algorithm 4). It is interesting to note that optimistic interferences only incur a relatively small increase in the number of \text{move} instructions while significantly reducing the complexity of the computation of the interference graph.

6 Conclusion

This paper presents a pinning-based solution to the problem of register coalescing during the out-of-SSA translation phase. We explain and demonstrate why considering \( \phi \) instruction replacement and renaming constraints together results in an improved coalescing of variables, thus reducing the number of \text{move} instructions before instruction scheduling and register allocation. We show the superiority of our approach both in terms of compile time and number of copies compared to solutions composed of existing algorithms (Sreedhar et al., Leung and George, Briggs et al., repeated register coalescing). These experiments also show that the affinity and interference graphs are usually quite simple, which means that a global optimization scheme would bring very little improvement over our local approach. Finally, we implemented some small variations of our algorithm, and observed that an optimistic implementation of interferences still provides good results with a significant reduction in the complexity of the computation of the interference graph. During this work, we also improved slightly the mark and reconstruct phases of Leung and George’s algorithm, which we rely on.

References


Algorithm 2 Formal description of our algorithm.

Create_affinity_graph(CFG_Node current_node)
\[ (E, V) = (\emptyset, \emptyset) \]
for each \( X = \phi(x_1, \ldots, x_n) \) of current_node
\[ V = V \cup \{ \text{Resource}_\text{def}(X) \} \]
for each \( x \in \{ x_1, \ldots, x_n \} \)
\[ V = V \cup \{ \text{Resource}_\text{def}(x) \} \]
\[ e = (\text{Resource}_\text{def}(X), \text{Resource}_\text{def}(x)) \]
if \( e \notin E \)
\[ \text{multiplicity}(e) = 0 \]
\[ E = E \cup \{ e \}, \text{multiplicity}(e)++ \]
return \( G = (E, V) \)

Graph_InitialPruning(Graph \( (V, E) \))
foreach \((x_1, x_2) \in E, \)
if \( (\text{Resource}_\text{interfere}(x_1, x_2)) \)
\[ E := (x_1, x_2) \]
return \( (V, E) \)

BipartiteGraph_pruning(Bipartite_Multi_Graph \( (V, E) \))
{ Evaluates the weight for each edge }
for all \( e \in E \), \( \text{weight}(e) = 0 \)
for all \( (x, x') \in E^2 \) such that \( x \neq x' \)
if \( \text{Resource}_\text{interfere}(x, x') \)
\[ \text{weight}((x, x')) = \text{multiplicity}((x, x')) \]
\[ \text{weight}((x, x')) = \text{multiplicity}((x, x')) \]
{ Prunes in decreasing weight order and update the weight }
while \( \text{weight}((e_p)) > 0 \)
let \( e_p = (x, y) \) such that \( \forall e \in E, \text{weight}(e_p) \geq \text{weight}(e) \)
do
\[ E := e_p \]
for all \( e = (x, y) \in E \)
\[ \text{weight}(e) := \text{multiplicity}(e_p) \]
for all \( e = (y, x) \in E \)
\[ \text{weight}(e) := \text{multiplicity}(e_p) \]
return \( (V, E) \)

PrunedGraph_pinning(Graph \( G , \) Program \( P \))
foreach \( V \in \{ \text{connected components of } G \} \)
let \( u = \bigcup_{V \in V} V \)
let \( w = \left\{ \begin{array}{ll} v_i & \text{if } v_i \in V \text{ is a physical resource} \\ \emptyset & \text{otherwise} \end{array} \right. \)
foreach \( (\text{OP})d_1, \ldots = \text{instr}(a_1, \ldots) \in P \)
foreach \( d_i \) such that \( d_i \in u \)
pin \( d_i \) to \( w \) in \( (\text{OP}) \)
foreach \( a_i, \gamma \) such that \( r \in V \)
replace \( r \) by \( w \)

Variable_kills(Variable \( a \), Variable \( b \))
if the definition of \( b \) dominates those of \( a \)
and \( a \) and \( b \) interfere
return true (Case 1)
if \( a \) is defined as \( a = \phi(a_1 : B_1, \ldots, a_n : B_n) \)
for \( i = 1 \) to \( n \)
if \( b \) is live out of \( B_i \) and \( b \neq a_i \)
return true (Case 2)
return false

Resource_killed(Resource \( A \))
let \( A = \{ a_1, \ldots, a_n \} \)
\[ \text{killed}_\text{within}A = \{ a \in A \exists a_j \in A, \text{Variable}_\text{kills}(a_j, a) \} \]
return \( \text{killed}_\text{within}A \)

Resource_interfere(Resource \( A \), Resource \( B \))
let \( A = \{ a_1, \ldots, a_n \} \)
let \( B = \{ b_1, \ldots, b_m \} \)
let \( \text{killed}_\text{within}A = \text{Resource}_\text{killed}(A) \)
let \( \text{killed}_\text{within}B = \text{Resource}_\text{killed}(B) \)
if \( A \) and \( B \) are physical resources
if \( A \neq B \) return true
for all \((a, b) \in A \times B \)
if \( a \notin \text{killed}_\text{within}A \) and \( \text{Variable}_\text{kills}(b, a) \)
return true
if \( b \notin \text{killed}_\text{within}B \) and \( \text{Variable}_\text{kills}(a, b) \)
return true
if \( \text{Variable}_\text{stronglyInterfere}(a, b) \)
return true
return false
Algorithm 3 Construction of initial affinity graph with a depth constraint.

\begin{algorithm}
\caption{Create\_affinity\_graph(CFG\_Node current\_node, Integer depth)}
\begin{algorithmic}
\State $(E, V) = (\emptyset, \emptyset)$
\For {each $X = \phi(x_1, \ldots, x_n)$ of current\_node}
\State $V = V \cup \{\text{Resource}\_\text{def}(X)\}$
\For {each $x \in \{x_1, \ldots, x_n\}$}
\State let Node\_x: $x = \ldots$
\If {depth(Node\_x) $\neq$ depth}
\State continue
\EndIf
\State $V = V \cup \{\text{Resource}\_\text{def}(x)\}$
\EndFor
\State $e = (\text{Resource}\_\text{def}(X), \text{Resource}\_\text{def}(x))$
\If {($e \notin E$) multiplicity($e$)=0}
\State $E = E \cup \{e\}$, multiplicity($e$)++
\EndIf
\EndFor
\EndFor
\State return $G = (E, V)$
\end{algorithmic}
\end{algorithm}

Algorithm 4 Optimistic and pessimistic definition of interferences.

\begin{algorithm}
\caption{Variable\_kills\_optimistic(Variable $a$, Variable $b$)}
\begin{algorithmic}
\State let Node\_a: (Def\_a) $a = \ldots$
\State let Node\_b: (Def\_b) $b = \ldots$
\If {($a \neq b$) and (Def\_b dominates Def\_a) and $b \in \text{liveout(Node\_a)}$}
\State return true \{Case 1\}
\EndIf
\If {a is defined as $a = \phi(a_1 : B_1, \ldots, a_n : B_n)$}
\For {i = 1 to n}
\If {b is live out of $B_i$ and $b \neq a_i$}
\State return true \{Case 2\}
\EndIf
\EndFor
\EndIf
\State return false
\end{algorithmic}
\end{algorithm}

\begin{algorithm}
\caption{Variable\_kills\_pessimistic(Variable $a$, Variable $b$)}
\begin{algorithmic}
\State let Node\_a: (Def\_a) $a = \ldots$
\State let Node\_b: (Def\_b) $b = \ldots$
\If {($a \neq b$) and (Def\_b dominates Def\_a) and ($b \in \text{livein(Node\_a)}$) or (Node\_a = Node\_b))}
\State return true \{Case 1\}
\EndIf
\If {a is defined as $a = \phi(a_1 : B_1, \ldots, a_n : B_n)$}
\For {i = 1 to n}
\If {b is live out of $B_i$ and $b \neq a_i$}
\State return true \{Case 2\}
\EndIf
\EndFor
\EndIf
\State return false
\end{algorithmic}
\end{algorithm}
Appendix A: Limitations and refinement of Leung’s algorithm

Once pinning has been performed, our algorithm relies on Leung’s mark and reconstruct algorithms to restore the code into non-SSA form. Critical edges are subject to a particular treatment in Leung’s algorithm. But as illustrated by Figure 14, the solution is not robust enough when dealing with aggressive pinning. The goal of this appendix is to propose a clearest semantic for $\phi$ functions, and to modify Leung’s algorithm accordingly.

To begin with, let us consider a $\phi$ definition $B : y = \phi(\ldots)$. The semantic used by Leung et al. is that this definition is distributed over each predecessor of $B$. Hence, in a certain sense multiple definitions of $Y$ coexist and therefore may conflict. Because conflicts for simple variables (not resources) are not taken into account in Leung’s algorithm, the lost copy problem has a special treatment that corresponds to the lines below the “(*fix problem related to critical edge*)”. Here, the copy $cp_2 := cp_2 \cup \{ w \leftarrow z \}$ is incorrect (probably a typo) and it is difficult to fix to obtain a correct and efficient code. Instead, that whenever the definition of $y$ is not pinned to any resource, we propose to create a virtual resource $y$ and to pin this definition to it. This fix takes place in the COLLECT procedure.

Another consequence of Leung’s $\phi$ function semantic is that whenever $y$ has to be repaired, the repairing copies are also distributed over each predecessors of $B$. Hence conflicts can occur and those repairing copies cannot be used further $B$ (which explains the need to introduce another repairing copy $w$). Because we found no a priori motivation to do so, we propose to place the repairing copy of $y$ just after its definition instead. Hence, our new semantic of a given $B : y \uparrow^r = \phi(x_1 : B_1, \ldots, x_n : B_n)$ (where $y$ is always pinned to a resource $r$) definition is the following:

- at the end of each block $B_i$, there is a new virtual instruction that defines no variable but that uses $x_i \uparrow^r$.
- at the beginning of the block $B$, the $\phi$ instruction contains no use arguments, but defines $y \uparrow^r$.
- all the “virtual uses” of the end of each block have a parallel semantic i.e. are considered all together.

The consequence is a simplification of the code: whenever instructions of a block have to be traversed then $\phi$ functions definitions, normal instructions uses, normal instructions definitions and $\phi$ functions uses (of each successors) are considered consecutively.

Figure 14: The $\phi$-function replacement conflict problem
The refined code is given below, modified code is written using the \( \triangleright \) sign.

Finally, we would like to outline the problem with dedicated register pinning. Indeed, we could find in Leung’s collect phase the code “if \( y \) was renamed from some dedicated register \( r \) during SSA construction then \( \text{must def}[y] = r \)...”. As illustrated by Figure 15 copy-folding performed on dedicated register definition can lead to an incorrect pinning. Because of its non local property, this inconsistency is not trivial to detect while doing the optimization. Freezing optimizations when dealing with dedicated registers is a solution to this problem. On the other hand the semantic is not necessarily strict enough to justify such a decision and pinning may be performed correctly while being aware of this specific semantic. Hence because dedicated registers related pinning that is semantic aware can be very complex, we have intentionally removed this part from the COLLECT procedure and delegated it to a previous pinning phase.

Figure 15: The \textit{parallel-copies conflict problem} is generated by a too constrained pinning.
procedure COLLECT
    initialize all entries of must_def and must_use to ⊥
    R := ∅
    for b ∈ basic blocks do
        for i ∈ φ-functions in b do
            if pinned_def(i, 1) then
                then let r be the dedicated register required
            else let r = y
                must_def[y] := r
                R := R ∪ {r}
        for i ∈ non-φ-functions in b do
            let i ≡ y₁ . . . yₘ ← op(x₁ . . . xₙ)
            for j := 1 to m do
                if pinned_def(i, j) then
                    let r be the dedicated register required
                    must_def[yᵢ] := r
                    R := R ∪ {r}
                for j := 1 to m do
                    if pinned_use(i, j) then
                        let r be the dedicated register required
                        must_use[i][j] := r
                        R := R ∪ {r}
    procedure MARKINIT
    for b ∈ basic blocks do
        sites[r] := ∅
        for r ∈ R do
            last[b][r] := ⊤
            phi[b][r] := ⊤
            for i ∈ φ-functions in block b do
                let i ≡ y ← φ(x₁ . . . xₙ)
                if must_def[y] = r ≢ ⊥ then
                    last[b][r] := xᵢ
                    sites[r] := sites[r] ∪ {b}
            for i ∈ normal instructions in block b do
                let i ≡ y₁ . . . yₘ ← op(x₁ . . . xₙ)
                for j := 1 to m do
                    if must_use[i][j] ≠ ⊥ then
                        last[b][r] := yᵢ
                        sites[r] := sites[r] ∪ {b}
                        for b’ ∈ succᵢᵢ(b) do
                            let b be the jth predecessor of b’
                            for i ∈ φ-functions in b’ do
                                let i ≡ y ← φ(x₁ . . . xₙ)
                                USE(i, j, xᵢ)
                                if must_use[i][j] ≠ ⊥ then
                                    avail[must_use[i][j]] := xᵢ
                                    in_place[i][j] = ∅
                                    if must_use[i][j] ≢ ⊥ and avail[must_use[i][j]] = x then
                                        in_place[i][j] = true
                                    return
                                if must_def[x] ≠ ⊥ and avail[must_def[x]] ≠ x then
                                    if repair_name[x] = ⊥ then
                                        repair_name[x] := a new SSA name
                                        repair_sites[x] := repair_sites[x] ∪ {i}
                                    if must_use[i][j] ≢ ⊥ then
                                        avail[must_use[i][j]] := xᵢ
                                    procedure DEFINE(x)
                                    if must_def[x] ≠ ⊥ then avail[must_def[x]] := xᵢ
                                    avin[b][r] =
            \begin{cases}
                x \text{ if } last[b][r] = x \neq ⊤ \\
                avin[b][r] \text{ otherwise}
            \end{cases}


procedure USE(i, j, x)
    in_place[i][j] = false
    if must_use[i][j] ≠ ⊥ and avail[must_use[i][j]] = x then
        in_place[i][j] = true
    return

if must_def[x] ≠ ⊥ and avail[must_def[x]] ≠ x then
    if repair_name[x] = ⊥ then
        repair_name[x] := a new SSA name
        repair_sites[x] := repair_sites[x] ∪ {i}
    if must_use[i][j] ≠ ⊥ then
        avail[must_use[i][j]] := xᵢ

procedure DEFINE(x)
    if must_def[x] ≠ ⊥ then avail[must_def[x]] := xᵢ
    avin[b][r] =
    \begin{cases}
        \top \text{ if } b \text{ is the entry} \\
        x \text{ if } phi[i][r] = x \neq top \\
        \bigwedge_{b' \in \text{pred}_{i, j}(b)} \text{avout}[b'][r] \text{ if } b \in DF⁺(sites[r]) \\
        \text{avout}[idom(b)][r] \text{ otherwise}
    \end{cases}
procedure LOOKUP(i, x)
  if stacks[x] is empty then
    if must_def[x] ≠ ⊥ then
      return must_def[x]
    else return x
  else
    let (y, sites) = top(stacks[x])
    if i ∈ sites then return y
    else if must_def[x] ≠ ⊥ then return must_def[x]
    else return x
  return x

procedure RENAME_USE(i, j, x, copies)
  let y = LOOKUP(i, x)
  let r = must_use[i][j]
  if inplace[i][j] then
    rewrite the jth input operand of i to r
  else if r ≠ ⊥ then
    copies := copies ∪ {r ← y}
    rewrite the jth input operand of i to r
  else
    rewrite the jth input operand of i to y
  return copies

procedure RENAME_DEF(i, j, y, copies)
  let r = if must_def[y] = ⊥ then y else must_def[y]
  rewrite the jth output operand y to r
  if repair_name[y] = tmp ≠ ⊥ then
    push (tmp, repair_sites[y]) onto stacks[y]
  copies := copies ∪ {tmp ← r}
  return copies

procedure RECONSTRUCT(b)
  for i ∈ φ-functions in b do
    let i ≡ y ← φ(x₁...xₙ)
    ▷ cp₄ = ∅
    ▷ cp₄ = RENAME_DEF(i, 1, y, cp₄)
    ▷ insert parallel copies cp₄ after i
  for i ∈ normal instructions in b do
    (* rewrite instructions *)
    let i ≡ y₁...yₘ ← op(x₁...xₙ)
    cp₁ := ∅
    for j := 1 to n do
      cp₁ := RENAME_USE(i, j, xⱼ, cp₁)
    insert parallel copies cp₁ before i
    cp₂ := ∅
    for j := 1 to m do
      cp₂ := RENAME_DEF(i, j, yⱼ, cp₂)
    insert parallel copies cp₂ after i
    cp₃ := ∅
    (* compute φ-copies *)
    for b′ ∈ succcfg(b) do
      let b be the kth predecessor of b′
      for i ∈ φ-functions in b′ do
        let i ≡ y ← φ(x₁...xₖ...xₙ)
      ▷ for j := 1 to n do
        cp₃ := RENAME_USE(i, j, xⱼ, cp₃)
      insert parallel copies cp₃ at the end of block b
    for b′ ∈ succdom(b) do
      RECONSTRUCT(b′)
      Restore stacks[] to its state
      at the beginning of this call
Appendix B: NP-completeness results

This appendix is devoted to the proof of LOCAL PINNING NP-completeness. Also, because this proof can be extended to the global problem GLOBAL PINNING the corresponding proof is provided. Remark that the result is valid with or without renaming constraints. For simplicity, proofs are made without renaming constraints. Also, additional remarks concerning the coalescing problem complexity in its general form are provided.

We start with a few definitions.

Definition 1 (GLOBAL PINNING) Consider a SSA program $P$ containing no initial pinning and a set of $\phi$ definitions $X_i = \phi(x_{i,1}, \ldots, x_{i,n_i})$. Let us denote by $DEFS = \{X_1, \ldots, X_n\}$, $ARGS_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$, $ARGS = \bigcup_i ARGS_i$ and $V = DEFS \cup ARGS$. Find a partitioning of $V$ into disjoint sets $R_1, \ldots, R_m$ such that

(CK): $\bigcup_{v \in V} \text{Resource}\_\text{killed}(\{v\}) = \bigcup_{1 \leq j \leq m} \text{Resource}\_\text{killed}(R_j)$ (no more killed variable)

(CS): $\forall 1 \leq j \leq m, \forall (x, y) \in R_j^2, \neg \text{Variable}\_\text{stronglyInterfere}(x, y)$ (no strong interference)

(CM): $\text{card} \left[ \left( \bigcup_{1 \leq i \leq n} DEFS \times ARGS_i \right) \cap \left( \bigcup_{1 \leq i \leq m} R_i^2 \right) \right]$ is maximized

Definition 2 (LOCAL PINNING) Consider a program $P$ with some pinning already performed and a set of $\phi$ definitions $X_i = \phi(x_{i,1}, \ldots, x_{i,n_i})$ within the same block $B$. Let us denote by $DEFS = \{X_1, \ldots, X_n\}$, $ARGS_i = \{x_{i,1}, \ldots, x_{i,n_i}\}$, $ARGS = \bigcup_i ARGS_i$, $V = DEFS \cup ARGS$ and $x$ the set of variables pinned to the same resource than $x \in V$. Find a partitioning of the set of resources $V$ into disjoint sets $R_1, \ldots, R_m$ such that

$\bigcup_{v \in V} \text{Resource}\_\text{killed}(v) = \bigcup_{1 \leq j \leq m} \text{Resource}\_\text{killed}(\bigcup R_j)$ (no more killed variable)

$\forall 1 \leq j \leq m, \forall (x, y) \in \bigcup R_j^2, \neg \text{Variable}\_\text{stronglyInterfere}(x, y)$ (no strong interference)

$\text{card} \left[ \left( \bigcup_{1 \leq i \leq n} DEFS \times ARGS_i \right) \cap \left( \bigcup_{1 \leq i \leq m} \bigcup R_i^2 \right) \right]$ is maximized

Theorem 1 GLOBAL PINNING is NP-complete in the size of $\phi$ functions.
Proof of Theorem 1 We prove the theorem using the polynomial reduction to M\textsc{aximum-Independent-Set}. Hence, let us consider a graph $G = (V, E)$ where $V = \{x_1, \ldots, x_n\}$. We aim to find a maximum independent set $I \subset V$ i.e.

\[
\begin{align*}
\text{card } I & \text{ is maximum} \\
\text{for each } (x_i, x_j) & \in I^2, (x_i, x_j) \not\in E
\end{align*}
\]

Let us build the corresponding instance of G\textsc{lobal Pinning}:

- For all $x_i \in V$, consider a block $Bi$ which contains a definition of $x_i$
- For all $(x_i, x_j) \in E$ consider
  1. two blocks $Bi_{ij}$ and $Bj_{ij}$ with predecessor $Bi$ and resp. $Bj$. A third block $B_{ij}$ with predecessors $Bi_{ij}$ and $Bj_{ij}$.
  2. a definition $a_{ij}$ in block $Bi_{ij}$ and a definition $ai_{ij}$ in block $Bj_{ij}$
  3. two $\phi$ functions in block $B_{ij}$:

\[
\begin{align*}
&Ci_{ij} = \phi(x_i, ai_{ij}) \\
&Cj_{ij} = \phi(a_{ij}, x_j)
\end{align*}
\]
- A block $B$, with predecessors $B1, \ldots, Bn$, which contains the instruction $X = \phi(x_1, \ldots, x_n)$

For this program,

- there is an affinity between $X$ and all $x_i$; for each $(x_i, x_j) \in E$ there are affinities between $Ci_{ij}$ and $x_i$ and between $Ci_{ij}$ and $ai_{ij}$
- interferences are between all couple $ai_{ij}$ and $x_j$ and between all couple $Ci_{ij}$ and $Cj_{ij}$.

Figure 16: Reduction for the NP-completeness proof of G\textsc{lobal Pinning}: $G'$ contains $|E_a| = 4 \times |E| + |V| = 11$ affinity edges.
On this program, \textsc{Global Pinning} can be seen as follows: find a correct coloring of the affinity-interference graph \((G' = (V', E_a, E_i))\) that maximizes the number of affinity edges (dashed edges: \(E_a\)) between two vertices of the same color. A coloring is correct whenever there is no interference (full edges: \(E_i\)) between two vertices of the same color. This new graph \(G'\) contains four affinity edges for each edge of the initial graph \(G\) \((4|E|\) on the whole), and one affinity edge for each vertex of \(G\) \(|V|\) on the whole). See Figure 16 for a simple example. Consider an optimal solution to \textsc{Global Pinning} with \(X\) the resource containing \(X\). \(I = X \cap V\) is the set of vertices of \(G\) that have the same color than \(X\). Let \((x_i, x_j) \in E\) and consider the corresponding four affinity edges. As illustrated by Figure 17 if \(x_i\) and \(x_j\) are in \(I\) (so they get the same color), from those four affinity edges only a maximum of two can be satisfied. Hence, the cost of this solution can be bounded by

\[
\hat{C}(I) \leq |I| + 4|E| - \sum_{(x_i, x_j) \in E \cap I} 2
\]

Now, consider a solution where \(I\) is an independent set and all other \(x_i\) have different colors, then by coloring remaining vertices as illustrated in Figure 17 the upper bound \(|I| + 4|E|\) is reached. Hence, the solution is optimal if and only if \(I\) is a maximum independent set.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure17}
\caption{\(I\) is an independent set.}
\end{figure}

\textbf{Theorem 2} \textsc{Local Pinning} is \textit{NP}-complete.

\textbf{Proof of Theorem 2} The proof uses the same reduction than for \textsc{Global Pinning}: for a given graph \(G = (V, E)\) we consider the same program and suppose that blocks \(B_{ij}\) have already been performed. Block \(B\) remains. At this stage we have \(\bar{x}_i = \{C_{ij}, x_i, a_{ij}\}\) for all \(i\) and \(\bar{X} = \{X\}\). Hence, \((x_i, x_j) \in E\) if and only if \(\bar{x}_i\) interfere with \(x_j\). So, the optimal partitioning of \(\{X, x_1, \ldots, x_n\}\) provides with \(\bar{X} \cap V\) an independent set for \(G\).
The **MAX-COALESCE problem is NP-complete** To finish with, let us define formally the coalescing problem (MAX-COALESCE): consider a program with virtual variables and move instructions between some of those variables. To get a stronger result, consider that this program has been obtained by a basic out-of-SSA translation i.e. no coalescing have been performed during this reconstruction phase. Consider the corresponding interference graph, and affinity graph: two variables interfere \(^2\) if they cannot be assigned to a common resource; two variables share an affinity if they are move related. Interferences are represented using full edges and affinities are represented using dashed edges. Affinity edges can be weighted to reflect the number of related move instructions and branch frequency prediction (e.g. [14]). To simplify the discussion, we consider that all blocks have an equal execution frequency, but the results given below are correct even if execution frequencies are taken into account.

Now, the coalescing problem consists on coloring correctly this graph using an unbounded number of colors. A correct coloring is a coloring such that two variables with the same color do not interfere. Variables within the same color-set are said to be coalesced. Each move instruction between two coalesced variables can be removed. The goal is to maximize the number of such move instructions, which exactly corresponds to maximizing the weighted number of dashed edges between coalesced variables. As illustrated by Figure 18, the previous NP-completeness proof can be easily modified to prove the NP-completeness of MAX-COALESCE problem.

\[ \text{[Initial Graph for MAX-INDEPENDANT-SET]} \quad \text{[Corresponding out-out-of SSA]} \quad \text{[program for MAX-COALESCE]} \quad \text{[Corresponding affinities] [and interferences in } G' \text{]} \]

Figure 18: Reduction for the NP-completeness proof of MAX-COALESCE: the corresponding affinity/interference graph is similar to the one obtained for the NP-completeness proof of GLOBAL PINNING.

The **MIN-MOVE problem is NP-complete** One can wonder if the complexity comes from the out-of-SSA algorithm itself, from the place of move instructions that replace \( \phi \) functions. In particular one can imagine performing code-motion and live-range splitting in order to get an even better result in term of number of move instructions. We name this problem MIN-MOVE. Unfortunately, the previous example with a similar reasoning also leads to the NP-completeness of the MIN-MOVE problem.

\(^2\)Remark that on our interference model \( X \) and \( x_1 \) of Figure 18 do not interfere simply because they share the same value whereas on some classical definitions they do. This can be easily overcomed by splitting edges between \( x_i \) and \( X \)