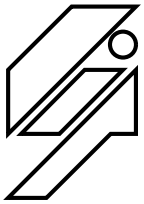
d-Dimensional Range Search on Multicomputers

Afonso Ferreira, Claire Kenyon, Andrew Rau-Chaplin, Stephane Ubeda



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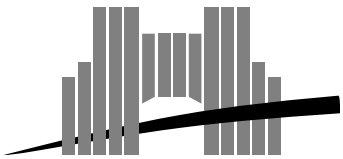

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Abstract

Given a set L of n points in the d-dimensional Cartesian space E^d, and a query specifying a domain q in E^d, the Range Search problem consists in identifying the subset $R(q)$ of the points of L contained in q.

The Range Tree data-structure represents a particularly good balance between storage space and search time. The structure requires $O(n \log^{d-1} n)$ space and construction time, but supports an $O(\log^d n)$ time search algorithm.

In this paper, we describe a set of efficient scalable algorithms for the construction and manipulation of a distributed analog to the sequential range tree data structure. We then show how to perform $O(n)$ independent range searches on a distributed range tree, in parallel.

These parallel construction and search algorithms are both optimal in the *Coarse Grained Multicomputer* model (also referred to as the weak-CREW BSP model), in the sense that their running times are the sequential time divided by the number of processors, plus a constant number of parallel communication rounds (i.e., h-relations in BSP context).

Keywords: Multidimensional search, Parallel algorithm, Distributed data structures, Coarse grained multicomputers

Résumé

Prenons un ensemble de n points dans un espace cartésien E^d, ainsi qu'une requête q définissant un domaine de E^d. On appelle *Range Search* le problème consistant à identifier le sous-ensemble $r(q)$ de points de L contenu dans q.

Le *Range Tree* est une structure de données offrant un excellent compromis entre la taille mémoire nécessaire à son stockage et le temps de réponse au problème. Il nécessite $O(n \log^{d-1} n)$ emplacements mémoire et permet de traiter une requête en $O(\log^d n)$ unités de temps.

Dans cet article, nous présentons des algorithmes efficaces et extensibles pour construire et manipuler une version distribuée du *Range Tree* comparable à la version séquentielle. Nous montrons comment répondre à $O(n)$ requêtes indépendantes en parallèle.

Les algorithmes de construction du *Range Tree* distribué et de réponses aux requêtes sont optimaux dans le modèle de machine à gros grains (aussi appelé modèle weak CREW-BSP), optimal signifiant que le temps d'exécution en parallèle est égal au temps d'exécution en séquentiel divisé par le nombre de processeurs, auquel on doit ajouter le temps nécessaire à un nombre constant d'étapes de communication (ou de *h-relations* dans le contexte du modèle BSP).

Mots-clés: Recherche multidimensionnelle, Algorithme parallèle, Structure de données distribuée, Machine parallèle à gros grain

d-Dimensional Range Search on Multicomputers*

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1 Introduction

The range tree is a fundamental data structure for multi-dimensional point sets, and as such, is central in a wide range of geometric and database applications[18]. The design and implementation of efficient parallel versions of this important data structure is one of the primary goals of this year's DIMACS Implementation Challenge[1]. In this paper, we describe the first non-trivial adaptation of range trees in the parallel distributed memory setting.

Our approach is to describe a set of efficient scalable algorithms for the construction and manipulation of a distributed analog to the sequential range tree data structure[4]. We then show how to perform $O(n)$ independent range searches on a distributed range tree T, in parallel. Note that the path of an individual search will trace in T is *not* known ahead of time, and must instead be determined "on-line". That is, only when a search query is at a node of T can it determine which node or nodes of T it should visit next. Also note that the paths of the search queries can overlap arbitrarily,

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such that any time any node of T may be visited by an arbitrary number of search queries.

The Model

Recently, there has been much interest in "realistic" parallel models (e.g. BSP, LogP, C^3, CGM) that can better predict the performance of parallel algorithms on existing, typically coarse or medium grained, parallel computers[21, 7, 16, 12]. In Valiant's BSP model, each communication round consists of routing a single arbitrary h-relation (i.e. each processor send and receives $O(h)$ data). Slackness in the number of processors is used to optimally simulate PRAM algorithms on distributed memory multicomputers. However, as Valiant points out, one may want to design "implementations of the BSP model that incorporate features for communications, computation or sychronization that are clearly additional to the ones in the definition"[21].

In this paper, we use the *Coarse Grained Multicomputer* model (CGM(s,p)), also sometimes referred to as the weak-CREW BSP model[15]. This model has been used (explicitly or implicitly) in parallel algorithm design for variety of problems [12, 15, 8, 11, 17, 9, 14] and has led to parallel codes exhibiting good timing results [12, 9, 14]. It consists of a set of p processors P_0 to P_{p-1} with $O(\frac{s}{p})$ local memory each, connected via some arbitrary interconnection network or a shared memory. The term "coarse grained" refers to the fact that the size of each local memory will typically be "considerably larger" than $O(1)$. We will assume $\frac{s}{p} \geq p$ as was assumed in [12], which is clearly true for all existing parallel machines. All algorithms consist of alternating local computation with global communications operations (Supersteps).

In this model, all global communications are performed by a small set of standard communications operations - Segmented broadcast, Segmented gather, All-to-All broadcast, Personalized All-to-All broadcast, Partial sum and Sort, which are typically efficiently realized in hardware. If a parallel machine does not provide these operations they can be implemented in terms of a constant number of sorting operations [12].

Moreover, recently Goodrich [15] has shown that, given $p < n^{1-\frac{1}{c}}$ ($c \geq 1$), sorting $O(n)$ elements distributed evenly over p processors in the BSP (or LogP) model can be achieved in $O(\log n / \log(h+1))$ communication rounds and $O(n \log n/p)$ local computation time, for $h = \Theta(\frac{n}{p})$, i.e. with optimal local computation and $O(1)$ h-relations, when $\frac{n}{p} \geq p$. Therefore, using this sort, the communication operations of the $CGM(s,p)$ can be realized in the

BSP (or LogP) models in a constant number of h-relations, where $h = \Theta(\frac{s}{p})$. Hence, in the remainder, any of the above global communication operations on the $CGM(s,p)$ will be denoted $T_c(s,p)$.

Finding an optimal algorithm in the CGM model is equivalent to minimizing the number of global communication rounds as well as the local computation time. It has been shown that minimizing the number of supersteps also results in improved portability across different parallel architectures[21, 22].

The Multidimensional Range Search Problem

Consider a collection L of n records, where each record l has a value $key(l)$ and is identified by an ordered d-tuple $(x_1(l), ..., x_d(l)) \in E^d$, the d-dimensional Cartesian space. In the *orthogonal range search* problem, the query specifies a domain q in E^d, and the outcome of the search, depending on the application, may be either the subset $R(q)$ of the points of L contained in q, or the number of such points, or more generally a function $\bigotimes_{l \in R(q)} f(l)$, where $f(l)$ is an element of a commutative semigroup with operation $\otimes$. The former version of this problem is called the *report mode* while the latter version is called the *associative-function mode*. For lower bounds see [6].

There are many sequential data-structures and algorithms for range searching, each offering a different trade-off between storage and time complexity. These structures include k-D trees, multidimensional trees, Super-B trees, range trees, and layered range trees.

Multidimensional binary trees, commonly known as k-D trees are an optimal space solution, requiring $\Theta(dn)$ space, but having a discouraging worst-case search performance of $O(dn^{1-1/d})$ time [18]. Parallel algorithms for the range-search problem based on k-D trees have been studied for the scan computation model [5].

The Range Tree data-structure represents a particularly good balance between storage space and search time. The structure requires $O(n \log^{d-1} n)$ space and construction time, but supports an $O(\log^d n)$ time search algorithm [18]. An improved version of this structure, known as the layered range tree, saves a factor of $\log n$ in the search time. A parallel version of the range tree data structure was introduced for the SIMD hypercube model of computation [19]. It required $O(d \log n)$ search time per query using $O(\log^d n)$ processors. But, the parallelization scheme was based on copying of the data structure onto each processor, therefore requiring $O(pn \log^d n)$ memory space in total which is, in most situations, quite unrealistic. In

SODA'95, a derivative of the range tree data structure for secondary memory was described [20]. The one dimensional range search problem is solve in [13].

Our Results

Given a set L of n points in d-dimensional Cartesian space, we show how to construct on a CGM(s,p) a distributed range tree T in time $O(\frac{s}{p} + T_c(s))$, where $s = n \log^{d-1} n$ is the size of the sequential data structure. We then show how T can be used to answer a given set Q of $m = O(n)$ range queries in time $\mathrm{O}(\frac{s \log n}{p} + T_c(s,p))$ and $\mathrm{O}(\frac{s \log n}{p} + T_c(s,p) + \frac{k}{p})$, for the associative-function[1] and report modes respectively, where k is the number of results to be reported.

These parallel construction and search algorithms are both optimal, in the sense that their running times are the sequential time divided by the number of processors, plus a constant number of parallel communication rounds (i.e., h-relations with $h = \Theta(s/p)$).

Our solution is, in part, based on the Multisearch paradigm first introduced in [10] and later used to solve a variety of problems[12, 10, 3, 2]. It represents a significant advancement over the multisearch method described in [12] in that the lower dimensional substructures pointed to by each node of T is of non-constant size and queries that must visit several neighbours of a node of T can do so by "splitting" into several subqueries.

In very broad terms, our techniques for solving the range search problem are a judicious combination of the following ideas:

- Partition T into pieces (of differing shapes and dimensions), some of which are processed sequentially, while others are processed in parallel.
- Create multiple copies of those pieces of T for which too many searches need access, and distribute the copies to processors, each of which is responsible for advancing a manageable subset of the "congested" searches. It should be noted that the straightforward strategy of making multiple copies of T, and using one copy for each $\frac{n}{p}$ group of queries, does not work. This is due to the fact that it would not only take too much time to create the p copies, but there is not enough space to store all of these copies of T.
- Implement searches using multiple queries by, when necessary, making copies of those queries visiting a node v of T who require access to

[1] In the special case of associative functions with inverses, this problem can be solved using weighted dominant counting [12]

more than a single neighbour of v. Some of these queries are advanced sequentially, while others are advanced in parallel.

Of course, the parameters needed to efficiently perform these partitioning, duplication and mapping strategies cannot be precomputed, since the full search paths are computed on-line. Therefore, these parameters must also be determined on-line, as the searches advance through T. The above description is necessarily an over simplification, only a careful look at the details can reveal the exact interplay between the above ideas, as well as the exact nature of each of them.

The organization of the paper is as follows. Section 2 describes the classical range tree and our distribution scheme on p processors. A coarse grained parallel algorithm to build this distributed data structure is described in Section 3. Section 4 gives a coarse grained parallel algorithm to solve n queries in parallel with the distributed range tree.

2 On Range Trees

In this section we first define the segment tree and range tree data structures to be used in the remainder. Then, we define a labeling of the nodes of the range tree, in order to be able to store it efficiently in a distributed memory setting. Finally, we define the "hat" of such a structure, which is fundamental to our partitioning strategy.

2.1 Recall of Basic Definitions

Let a $(1, n)$ segment tree [4] be a complete rooted binary tree with n leaves. Each node is associated with a segment. The segments associated to the leaves are $[1..2[$, $[2..3[$,...,$[(n-1)..n[$ and $[n, n]$ (the last segment is reduced to a point). Each internal node is associated with the segment formed by the union of the two segments associated to its children. Thus, the segment associated with the root is $[1..n]$.

As presented in [18], the range tree structure is a generalization of the segment tree. Let each element of L be a point $l = (x_1(l), x_2(l), \ldots, x_d(l))$, $j = d$, and $L_j = L$. Finally, a segment tree is said to be *in dimension i* if the segments associated to its leaves are obtained by a projection of a subset of L onto dimension i.

Definition 1 *The j-dimensional range tree T for a set L_j is recursively defined as follows.*

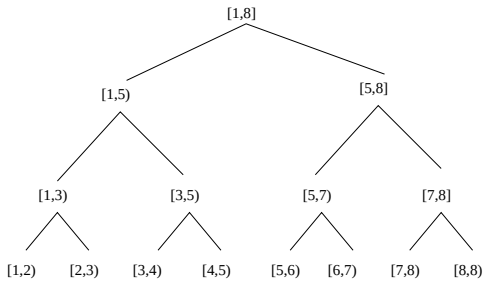

Figure 1: The segment tree structure for (1,8).

i *A primary segment tree T^* in dimension $d-j+1$ corresponding to the set $\{x_{d-j+1}(l)|l \in L_j\}$. For each node v of T^*, let $W(v)$ denote the set of points such that $x_{d-j+1}(l)$ lies in the interval associated with v. Define the $(j-1)$-dimensional set*

$$L_{j-1}(v) = \{(x_{d-j+2}(l), ..., x_d(l))|p \in W(v)\}.$$

ii *Each node v of T^* has a pointer to a range tree for $L_{j-1}(v)$ which is called descendent(v). For each node w in the primary segment tree of descendent(v), we define ancestor(w) $= v$.*

2.2 Labeling

To each node v of the range tree, we associate a unique label denoted path(v) which enables us to refer to nodes and to subtrees of T, which is defined as follows.

Definition 2 *For any node v of a range tree we define the following indices.*

i *Level(v) is the length of the shortest path from v to a leaf (or 0 if v is a leaf).*

ii *Index(v) =*

- *0 if v is the root of T*
- *Index(ancestor(v)) if v is a root of any segment tree except T*
- *2×Index(parent(v)) if v is a left child in a segment tree.*
- *2×Index(parent(v))+1 if v is a right child in a segment tree.*

(see Fig. 2)

iii *Path-index(v) = ⟨index(v),level(v)⟩*

iv *Path(v) =*

- *path-index(v) if v is a node of T^**
- *⟨path-index(v),path(ancestor(v))⟩, otherwise.*

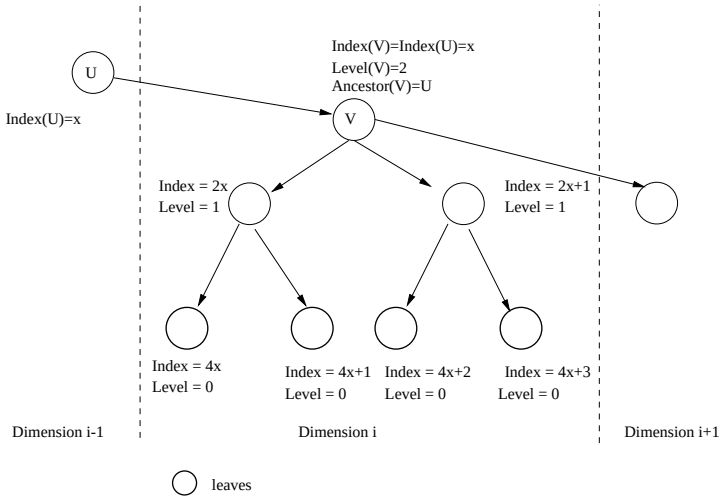

Figure 2: Illustration of $Index$ and $Level$ of a node of T.

Lemma 1 *For every segment tree $t \in F$ and all nodes $v \in t$, path(ancestor(v)) uniquely identifies the tree t to which v belongs.*

Proof: It is easy to see that for all nodes $v \in T$, path(v) is unique. Furthermore, it follows from Definition 1 that for every segment tree $t \in T$ and each pair of nodes $u, v \in t$, it holds that ancestor(u) = ancestor(v). Hence, path$(ancestor(u))$ = path(ancestor(v)) and this can be interpreted as the name of the segment tree t. □

2.3 The "hat"

A range tree T for a set of n points is of size $s = O(n \log^{d-1} n)$ [18], which is as large as the total memory available on our CGM(s, p). Therefore, the range tree must be partitioned into substructures where each substructure is of size $O(s/p)$. To support an efficient search strategy, some of these substructures will be stored on a single processor while others will be copied on to all processors such that each processor stores no more than $O(1)$ such structures.

Definition 3 *Given a range tree T,*

i *Let the "hat" H of T denote the subtree of T induced by all nodes v of T in the top* $\log p$ *levels, i.e. with level(v)* $\geq \log n - \log p$.

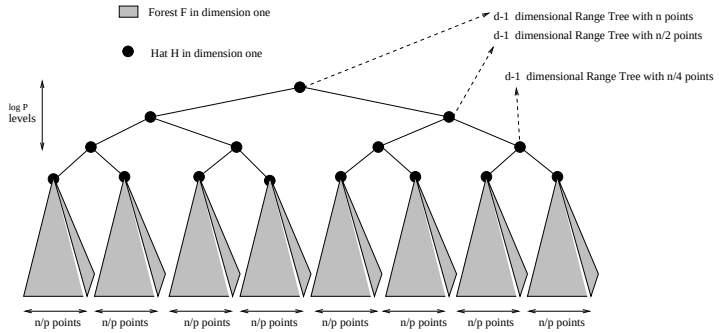

Figure 3: The hat of T in dimension 1, along with the associated part of F, for $p = 8$.

ii *Let F denote the forest of subtrees of T whose roots are the leaves of the hat H, i.e. the subtrees induced by all nodes v of T with level(v) $\leq \log n - \log p$. Note that each element of this forest is a range-tree on n/p points and has dimension $j \in [1..d]$ (see Fig. 3)*[2].

iii *For each range tree t in F with root r, let location(t)= i, where i is the index of the leaf of H corresponding to r. Note that such indexes are in the range $0 \ldots p-1$. Let $F_i = \{t \in F | location(t) = i\}$.*

Theorem 1 *The following holds for H and F_i as defined above.*

i) *The hat H has size $O(p \log^{d-1} p) = O(s/p)$.*

ii) *For every i, F_i has size $O(s/p)$.*

Proof:

i) Immediate from the fact that the hat is a range-tree with p leaves.

ii) For each $i \in [0..p-1]$, F_i consists of a set of range trees of various dimension (from 1 to d) of n/p points. By definition, the sets F_i are disjoint and have equal size, yielding $|F_i| = O(s/p)$, since the total data size is $O(s)$.

□

[2]In the one-dimensional case, where the range tree is just a segment tree, the hat consists of the top $\log p$ levels of the tree and the forest consists of the p subtrees rooted at level $\log p$.

3 Constructing a Distributed Range Tree

Our range search algorithm is based on a distributed representation of a range tree. The size of the range tree data structure for n items is $s = n \log^{d-1} n$, therefore we will use a p processor coarse grained multicomputer with $O(\frac{n \log^{d-1} n}{p})$ memory per processor, i.e. CGM(s, p). Without lost of generality, we assume (as in [18]) that all coordinates, in each dimension, are normalized by replacing each of them by their rank in increasing order (i.e. points are in $\{1, \ldots, n\}$) and that $n = 2^k$.

In the following, our distributed range tree will be stored on a CGM(s, p) as follows.

- A copy of the hat H will be stored on every processor and used as an index structure for the forest F.
- Each range tree t in F_i will be stored on processor P_i.

As seen in the previous section, both H and the F_i fit in a single processors memory.

In the following we describe a parallel algorithm for constructing the previously defined distributed range tree. As shown in [18], there exists an optimal sequential algorithm to build a d-dimensional range tree of size $O(n \log^{d-1} n)$, running in time $O(n \log^{d-1} n)$. This algorithm works in a bottom-up fashion in which segment trees are built up from their leaves one dimension after another.

As in sequential, the distributed range tree is constructed in d phases from phase $j = 1$ to phase $j = d$. At the start of phase j, let S^j be a set of records representing leaves of segment trees of T in dimension j. These segment trees must be constructed. More precisely, a record in S^j, corresponding to a point l from the original point set L, consists of two vectors: $l = \{x_1(l), \ldots, x_d(l)\}$ and a label index(l). This record in S^j is to become a leaf of the segment tree in T which is uniquely identified by index(l).

In phase j we first perform the data distribution: S^j will be sorted such that leaves of segment trees $t \in T$ with location$(t) = i$ are routed to processor P_i. Elements of F can then be sequentially constructed. Since the roots of the segment trees in F are the leaves of H, it suffices to perform an all-to-all broadcast of these roots in order to have all information required to complete the construction of the segment trees $t \in H$ in dimension j. Then, the set S^{j+1} is constructed.

The algorithm is as follows.

Algorithm Construct

Input: Each processor P_i stores a set I_i of n/p points drawn from L, arbitrarily.
Output: Each processor P_i stores
i) A copy of H;
ii) The set F_i.

0 Each processor creates for each element l of I_i an initial record e of S^1 with $e = (x_1(l), ..., x_d(l))$ and index(e)= nil. Let $j \leftarrow 1$.
1 Globally sort S^j by primary key index and secondary key x_j.
2 Each processor receives an ordered set of records from S^j representing leaves of trees $t \in F$ which are in dimension j. These trees must now be routed to the correct location. Each processor P_i divides its set into groups of n/p consecutive records, computes the global rank of each group and routes the k^{th} group to processor $P_{k \bmod p}$, using global sort.
3 Each processor P_i constructs sequentially the elements of F_i. Since the roots of F correspond to the leaves of H, all processors perform an all-to-all broadcast of the roots of their F_i.
4 Each processor receives $O(p \log^{d-1} p)$ roots and constructs its own copy of segment trees of H in dimension j.
5 if $j = d$ then exit.
6 Each record $z \in S^j$ stored in processor P_i belongs to a segment associated with a leaf y of H in dimension j (corresponding to the root of z's tree $t \in F$). In each processor P_i, for all z, walk from y's parent to the root of y's segment tree and for each node u visited create a new element s of S^{j+1} as follows: $x(s) = x(z)$ and index(s)=path(u).
7 $j \leftarrow j + 1$. Goto step **1**.

The correctness of Algorithm Construct follows from the sequential construction algorithm in [18], Definitions 1 and 3, Lemma 1, and Theorem 1. Its time complexity is $O(\frac{s}{p} + T_{sort}(s,p))$ where the first term comes from Steps 0, 3, 4 and 6, and the second from Steps 1, 2, and 3.

We thus have the following result.

Theorem 2 *A distributed range tree T can be constructed on a CGM(s,p) in time $O(\frac{s}{p} + T_c(s,p))$.*

This theorem and the weak-CREW BSP sorting algorithm from [15] imply the following.

Corollary 1 *A distributed range tree T can be constructed on a weak-CREW BSP in a constant number of h-relations ($h = \Theta(s/p)$) and $O(s/p)$ local computation time.*

4 Parallel Range Search

As presented in the Introduction, the parallel range search problem consists of answering the set Q of $m = O(n)$ range queries in parallel. In [18], an $O(\log^d n)$ sequential algorithm to solve the single query problem is given. The sequential algorithm for a query q on a range tree T runs as follows. Initially, q *visits* the root of T. When a query visits a node v in dimension j of T, it compares the query in the j^{th} dimension to the interval associated with v. There are four cases.

1. If the two segments are equal and $j < d$ then proceed to the next dimension, and the next node to be visited is the root of descendent(v).
2. If the two segments are equal and $j = d$ then v is the last node on q's search path and the segment tree rooted at v should be *selected* by q (i.e, all of its leaves are in the range of q).
3. If the two segments overlap (but are not equal), then the query q should be split into two queries: q', which is to visit the left child of v, and q'', which is to visit the right child of v.
4. If the two segments do not overlap the query q is deleted.

Note that each query q will visit at most $O(\log n)$ nodes in each dimension of T and $O(\log^d n)$ nodes will be selected in the final dimension d.

4.1 Identifying the results

The parallel algorithm for solving $m = O(n)$ queries takes the same basic approach. Initially, each processor P_i stores a set Q_i of n/p queries drawn from Q arbitrarily, and a distributed range tree T as described in Section 3. Note that a query is ready to report its result only when it visits a segment tree in dimension d of a range tree.

Thus, each processor P_i advances its queries through its copy of the hat H. This set dealt with, some of these queries select segment trees in dimension d of H, while others need to continue in F. The queries that have not completed their search paths and the required elements of F are then evenly balanced such that each processor stores $O(\frac{s}{p})$ queries along with the range trees from F they require. Finally, the queries are sequentially advanced through elements of F until they select segment trees in dimension d.

In the following algorithm, let $\bar{Q}$ denote the queries which have selected a segment tree in dimension d.

Algorithm Search

Input: Each processor P_i stores a set Q_i of n/p queries drawn arbitrarily from Q and a distributed range tree T.

Output: For each query $q \in Q$, a set of selected segment trees in dimension d of T and whose leaves correspond to the points of L in q's domain. Each such selected segment tree is given by an element of $\bar{Q}$.

0 Each processor P_i, advances its queries Q_i through the hat H. The queries which have already selected a segment tree in dimension d of H are put in $\bar{Q}$. Let $\hat{Q}$ denote the remaining queries, which need to visit a node in F.

1 Let $\hat{Q}_{F_j}$ denote those queries wanting to visit a tree $t \in F_j$. Globally, compute $c(j) = |\hat{Q}_{F_j}|/(|\hat{Q}|/p)$.

2 Make $c(j)$ copies of F_j and distribute them evenly.

3 Redistribute $\hat{Q}$ evenly so that every query $q \in \hat{Q}$ is stored on a processor that also stores a copy of the element of F which q is visiting.

4 Each processor P_j thus receives a set of queries and performs the sequential algorithm to select the appropriate segment trees, and puts the corresponding queries in $\bar{Q}$, thus completing $\bar{Q}$.

The load balancing phase, implemented in Steps **1** through **3** evenly distributes queries and forests F_i, such that each processor has $O(1)$ copies of each, as proved in [12].

Therefore, the correctness of Algorithm Search follows from the sequential construction algorithm [18] and from [12]. The time complexity of Algorithm Search is $\mathrm{O}(\frac{s \log n}{p} + T_{sort}(s,p))$ where the first term comes from Steps 0, and 4, and the second from Steps 1, 2, and 3.

Theorem 3 *Given a set Q of $m = O(n)$ range queries and a distributed range tree T for a set L of $O(n)$ points in E^d, stored on a CGM(s,p). Each element of Q can identify the subset of points from L in its domain, in time $O(\frac{s \log n}{p} + T_c(s,p))$.*

As in the previous section, combining this result with the weak-CREW BSP sort presented in [15] we get:

Corollary 2 *Given a set Q of $m = O(n)$ range queries and a distributed range tree T for a set L of $O(n)$ points in E^d, stored on a weak-CREW, BSP. Each element of Q can identify the subset of points from L in its domain, in a constant number of h-relations ($h = \Theta(s/p)$) and $O(\frac{s \log n}{p})$ local computation time.*

4.2 Reporting the results

In the range search problem, the query specifies a domain q in E^d, and the outcome of the search depends on the application. It may be either the subset L^q of the points of L contained in q (the *report mode*), or the number of such points, or more generally a function $\bigotimes_{l \in L^q} f(l)$, where $f(l)$ is an element of a commutative semigroup with operation $\otimes$ (the *associative-function mode*).

In this section we describe algorithms for both the associative-function and report modes running in time $\mathrm{O}(\frac{s \log n}{p} + T_c(s,p))$ and $\mathrm{O}(\frac{s \log n}{p} + T_c(s,p) + \frac{k}{p})$, respectively, where k is the number of results to be reported.

Algorithm Associative-Function

Input: A distributed range tree T, an associative function f, and a set Q of n queries.

Output: $f(q)$ for each query $q \in Q$.

0 Compute $f(v)$ bottom-up for each node v in dimension d of T as follows:
- Compute $f(v)$ for each node in trees of F in dimension d sequentially.
- All-to-all broadcast the values of $f(v)$ for each root of trees of F in dimension d.
- Compute $f(v)$ for each node v of the hat H in dimension d.

1 Perform Algorithm Search.

2 For each $q' \in \bar{Q}$, we create the pair (q, f(root of selected segment tree)).

3 Sort the pairs according to their first coordinate q.

4 For each block of pairs sharing a common q, compute f over the whole block (using a segmented partial sum).

Once we have the output of the Algorithm Search, it only remains to report the leaves of each selected segment tree. In order to do this in a balanced manner, we weigh the selected segment trees according to their sizes and redistribute them evenly, using again the load balancing procedure from [12].

Algorithm Report

Input: A distributed range tree T and a set Q of n queries.
Output: For each $q \in Q$ and each $l \in L$ in q's range, the pair (q, l) is on some processor.

0 Perform Algorithm Search to obtain a set of queries $q' \in \bar{Q}_i$ which have, each, selected segment trees in dimension d of T.
1 Compute for all $q \in \bar{Q}$ having selected a segment tree $t \in T$, the weight $w(q) = 2^{\text{level}(\text{root}(t))} =$ number of leaves of t.
2 Sort the elements of $\bar{Q}$ by weight.
3 Compute the partial sum $psw(q)$ for the element q of $\bar{Q}$ with respect to the weight $w(.)$, and let $dest(q) = p\lfloor psw(q)/\sum_{\bar{Q}} w(q) \rfloor$. Perform a segmented broadcast with destination $dest(.)$.
4 Make $w(q)$ copies of each query q and add it to $\bar{Q}$, associating with each copy a path to a leaf of the selected segment tree t. Each such copy corresponds to a pair (query of Q, point of L in q's range).

It is clear that algorithms Associative-Function and Report use only sequential procedures and the load balancing technique from [12]. Therefore,

Theorem 4 *Given a set Q of $m = O(n)$ range queries and a distributed range tree T for a set L of $O(n)$ points in E^d, stored on a CGM(s,p). All queries can be answered in both the associative-function and report modes in times $O(\frac{s \log n}{p} + T_c(s,p))$ and $O(\frac{s \log n}{p} + T_c(s,p) + \frac{k}{p})$, respectively, where k is the number of results to be reported.*

Again, considering the weak-CREW BSP sort presented in [15] we get:

Corollary 3 *Given a set Q of $m = O(n)$ range queries and a distributed range tree T for a set L of $O(n)$ points in E^d, stored on a weak-CREW BSP. Each element of Q can identify the subset of points from L in its domain, in a constant number of h-relations ($h = \Theta(s/p)$) and $O(\frac{s \log n}{p} + \frac{k}{p})$ internal computation time.*

5 Conclusion

In this paper, we defined a distributed range tree, the first non-trivial adaptation of range trees in the coarse-grained multicomputer model. We use this data structure to perform batched range search operations, in associative-function or in report mode, in optimal time. Our algorithms for constructing and searching the distributed range tree are a combination of standard communication primitives (such as parallel sort, used as a black box) and

of standard sequential range tree operations, so that the implementation on any variety of multicomputer should be relatively easy for a range tree expert.

Nonetheless, here we should inject a *caveat*: first, the construction algorithm is not quite optimal, since it uses parallel sort operations on sets of size $n \log^{d-1} p$, the number of leaves of the range tree, while ideally we would only wish to sort sets of size at most n, the number of input points. Second, there are some issues which need to be addressed in applications, in particular retrieving the answers to the queries in the report mode. However, we must stress that there currently is no viable alternative to the distributed range tree when the database is large enough to require a distributed data structure.

Finally, there are many issues still open. One is that the range tree is inherently static; a dynamic distributed data structure would be more powerful, although more difficult to implement. Another is that answering queries in batches of size n may be unsatisfactory in some applications, where n is very large. The question of using parallelism to speed up just one single query (or a few queries) is also wide open. This is open even in the much simpler case of segment trees, and would be worth studying.

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